Monetary Policy, Interest Rates, and Liquidity Premia[1]

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Abstract

The fact that central banks supply money in exchange for eligible assets, e.g. short-term treasuries, is typically neglected in macroeconomics. We augment an otherwise standard macroeconomic model and show that a liquidity premium emerges, when agents take the eligibility of assets into account. Consistent with empirical evidence, the model implies a negative relation between corporate bond yield spreads and the amount of available treasuries. The liquidity premium further provides a structural explanation for the systematic wedge between the policy rate and the marginal rate of intertemporal substitution that standard models equate. While policy rate changes are incompletely passed through to rates of return on non-eligible assets, monetary policy nevertheless exerts conventional effects on real activity and inflation.

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1   Introduction

Should macroeconomists care about monetary policy implementation? In macroeconomic models, (conventional) monetary policy is usually summarized by central banks controlling a short-term nominal interest rate of an asset that is illiquid until maturity. Yet, it is the rate on short-term treasuries, which typically serve as collateral for various financial market transactions, that closely relates to the expected policy rate, e.g. the Federal Funds rate,[4] while assets that are less liquid are priced differently.[5] The corresponding spreads between the treasury rate and the return on less liquid assets are non-negligible and can vary with the supply of treasuries (see Friedman and Kuttner, 1998, Longstaff, 2004, and Krishnamurthy and Vissing-Jorgensen, 2012). This paper presents a simple macroeconomic model with open market operations that accounts for these two observations. We show that considering central bank operations can lead to a liquidity premium on eligible assets (like treasuries),[6] when investors take collateral requirements of open market operations into account.[7] The model explains the observation that the marginal rate of intertemporal substitution systematically deviates from the rate set by the central bank, which has been identified as a major failure of standard macroeconomic models that equate both rates (see Canzoneri et al. 2007, Chari et al., 2009, and Atkeson and Kehoe 2009). We show that interest rates are affected by monetary policy consistent with empirical evidence, and we demonstrate that endogenous liquidity premia change the monetary transmission mechanism.

We present a simple framework which differs from standard macroeconomic models by specifying money supply as an explicit exchange of assets, which is usually neglected in the macroeconomic literature (see e.g. Smets and Wouters, 2007). The central bank is assumed to supply money solely against eligible assets, while private agents internalize the eligibility of assets when they invest. In equilibrium, the interest rate on an eligible asset, i.e. short-term treasuries, closely follows the discount rate the central bank charges when it purchases this asset, and it differs from interest rates on non-eligible assets by a liquidity premium. In equilibrium, this mechanism implies a negative correlation between the liquidity premium (based on corporate bond yield spreads) and the supply of treasuries, as recently documented by Krishnamurthy and Vissing-Jorgensen (2012). The model further provides a rationale for Canzoneri et al.'s (2007) and Atkeson and Kehoe's (2009) challenging finding that the spread between the rate implied by the consumption Euler

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[4]See e.g. Simon's (1990) or Lange et al.'s (2003) evidence from the treasury yield curve.

[5]How (il-)liquidity of assets affect their prices is, for example, shown by Holmstrom and Tirole (2001), Acharya and Pedersen (2005), or Lagos (2010).

[6]During the recent financial crises central banks have proven to be able to affect interest rates at various maturities by introducing lending facilities and direct asset purchases. See Fleming (2012) for an overview of the effects of US Federal Reserve liquidity provision policies in 2007-2009.

[7]The liquidity of assets has been considered for various purposes in Bansal and Coleman (1996), Canzoneri et al. (2008), Shi (2012), Kiyotaki and Moore (2012). Liquidity premia emerge in these studies because assets provide transaction services to different degrees, whereas monetary policy matters for asset prices in equilibrium rather than for the private agents' investment decision.

equation and the Federal Funds rate systematically varies with the monetary policy stance, as well as for the observation that this spread, which is also known as a "risk premium shock" (see Smets and Wouters, 2008), is more volatile than the rate itself that is implied by the consumption Euler equation (see Chari et al., 2009).[8]

The majority of macroeconomic studies on monetary policy focusses on a short-term interest rate as the central bank's operating target, which impacts on the private sector behavior by affecting intertemporal substitution, i.e. by relating the policy rate to inflation and consumption growth via the consumption Euler equation. The well-known failure of the consumption Euler equation to explain risk-free interest rates (see Weil, 1989) has typically been neglected in standard macroeconomics models, where the policy rate and the treasury rate are assumed to equal the Euler equation rate, i.e. the rate implied by the consumption Euler equation. Moreover, the Euler equation residual, i.e. the spread between the Euler equation rate and the observed policy rate, is typically more volatile than the Euler equation rate (see Chari et al., 2009) and is found to be negatively related to the observed policy rate (see Canzoneri et al., 2007, and Atkeson and Kehoe, 2009), which cast severe doubts on the assumed identity between the Euler equation rate and the policy in standard models. We show analytically and quantitatively that these observations can be reconciled within our macroeconomic model, where changes in the policy rate are – due to the liquidity premium – not one-for-one passed through to all short-term interest rates.[9] Moreover, we show that the spread between the yields on corporate debt and on treasuries is negatively related to the availability of treasuries, consistent with evidence for the US (see Friedman and Kuttner, 1998, Longstaff, 2004, and Krishnamurthy and Vissing-Jorgensen, 2012).

The model is specified in order to serve two main purposes: It is specified in a sufficiently rich way for a quantitative comparison of selected moments from US time series and from model generated series, and it allows for a closed-form solution (for specific parameter choices) to facilitate a transparent analysis of the main mechanism. In particular, demand for liquidity/cash is induced by a simple cash-in-advance constraint for the benchmark model, which is replaced by a money-in-the-utility-function specification to assess the robustness of the results (see section 5.4). We augment a standard sticky price model by two key assumptions: First, open market operations are separated from the asset market, where agents trade which each other and with the government. Before the asset market opens, private agents can acquire cash in open market operations from the central bank in exchange for eligible securities discounted with the policy rate. Eligible assets that are bought today can therefore be cashed in the next period, such that the rate of return on treasuries closely follows the expected future policy rate, which accords to empirical evidence,

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[8]Chari et al. (2009) suspect that this shock is "hardly likely to be invariant to monetary policy", which accords to the mechanism presented in this paper.

[9]Eisfeldt (2007) also argues that the demand for short-term treasury securities (T-bills) cannot solely be explained with consumption smoothing and suggests considering transactions demand of assets.

e.g., by Simon (1990). Second, we account for common central bank practice (like the Fed's or the BoE's in non-crisis times) and assume that only short-term treasuries are eligible in open market operations ("T-bills only"), while other assets – like equity or corporate bonds – are not accepted by the central bank.[10] Given that access to money relies on holdings of treasuries, private agents demand a higher return on non-eligible assets as a compensation for their illiquidity.[11] A higher policy rate then raises the price of money in terms of treasuries and therefore leads to a decline in the liquidity premium. The second assumption further implies that changes in the supply of treasuries can alter private agents' access to cash. An increase in the stock of treasuries (relative to real activity) thereby reduces the valuation of liquidity and thus the liquidity premium.

These assumptions imply a monetary transmission mechanism that differs from the way real activity is affected by monetary policy in standard macroeconomic models (e.g. Smets and Wouters, 2008), where the rate of intertemporal substitution equals the real policy rate. Consider, for example, an unexpected increase in the policy rate. Private agents, who are willing to hold both money and treasuries, then demand a higher treasury rate to be compensated for higher costs of acquiring new money in the next period, such that the treasury rate follows the expected future policy rate. Due to the cash constraint, aggregate demand positively depends on available means of payment, while access to the latter is constrained by the amount of eligible assets discounted with the current policy rate. A higher policy rate has, thereby, a contractionary effect on aggregate demand and inflation. Given that the liquidity premium falls, the increase in the real policy rate is not passed through one-for-one to real rates of return on non-eligible assets, like corporate debt or equity. Since monetary policy does not govern the rate of intertemporal substitution, changes in the policy rate affect consumption and investments to a smaller extent and in a different way than in standard models. For example, consumption habits are neither necessary nor sufficient for smooth consumption growth (unlike in standard models, see Fuhrer, 2000) and a reasonable amplitude of investment responses can already be generated by investment adjustment costs which are much smaller than suggested by estimates based on aggregate data (like in Smets and Wouters, 2007).[12]

The paper is organized as follows. Section 2 presents empirical evidence on interest rates and spreads. In Section 3, the model is developed. In Section 4, we provide analytical results on the behavior of interest rates and spreads. Section 5 presents quantitative results. Section 6 concludes.

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[10] This assumption accords to the Fed's asset aquisition policy before 2007 (see Federal Reserve Bank of New York, 2006). In 2006, for example, Treasury bills were the largest position accounting for one-third of the System Open Market Account (SOMA) holdings. Bills and Treasury coupon securities with a maturity below 2 years accounted for about two-third of SOMA holdings, while treasury securities of longer maturities and a relatively small amount of Treasury inflation-indexed securities completed the portfolio.

[11] To be more precise, there are two interest rate differentials due to the liquidity of assets: the spread between the rates of return on money and treasuries, and the spread between the rates of return on non-eligible assets and the treasury rate. Throughout the paper, we will focus on the latter.

[12] Our model suggest investment adjustment costs of a magnitude that is consistent with empirical evidence from disaggregate data (see Groth and Khan, 2011).

2 Empirical evidence

In this Section, we provide an empirical analysis on interest rates and spreads, which are interpreted as liquidity premia. In the first part of this Section, we analyze yield spreads involving treasury rates and their correlation with the amount of available treasuries, which relates to the analysis of Krishnamurthy and Vissing-Jorgensen (2012). The second part analyzes the spread between the Euler equation rate and the Federal Funds rate, relating to Canzoneri et al. (2007), Atkeson and Kehoe (2009), and Chari et al. (2009). The results presented in this Section indicate a substantial role for monetary policy in influencing interest rates and spreads, providing the point of departure for our theoretical analysis.

2.1 Interest rate on short-term treasuries

In a recent study, Krishnamurthy and Vissing-Jorgensen (2012) provide empirical evidence on the role of treasuries supply for corporate bond yield spreads. They find a negative relationship in US data between the supply of government debt and spreads between corporate and government debt yields.[13] Krishnamurthy and Vissing-Jorgensen (2012) argue that an increase in the supply of treasuries reduces their "convenience value", representing liquidity and safety attributes, and thereby reduce the corporate bond yield spread. For their baseline specification, they consider the spread between yields on AAA rated corporate bonds and yields on treasury bonds with a long maturity and the ratio of total government debt to GDP. Given that the focus of this paper is on the role of monetary policy on interest rate spreads, we examine slightly different variables.

When the Federal Reserve implements its interest rate target, it buys or sells assets against reserves in open market operations. In normal times, T-bills are the largest asset class held in the Federal Reserve portfolio as a result of open market operations.[14] To assess the particular role of monetary policy, we analyze the behavior of the spread between the 3-month high-grade commercial paper rate and the 3-month Treasury bill rate, which is also examined in Friedman and Kuttner (1998) and in Krishnamurthy and Vissing-Jorgensen's (2012) analysis of short-term interest rates. Using a structural asset pricing equation, Krishnamurthy and Vissing-Jorgensen estimate the impact of the total stock of treasury debt relative to GDP on that spread and argue that it reflects the price of liquidity, as there has never been a default on high grade commercial papers.[15] Given that this spread reflects liquidity (rather than safety) attributes, we will assess the plausibility of our model's predictions by comparing this spread with the model's counterpart.

To account for the argument that the valuation of treasuries should depend on the amount available to the private sector, we remove the Federal Reserve holdings of 3-month T-bills, to isolate

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[13] This result is related to Friedman and Kuttner's (1998) finding that the spread between commercial papers and treasury bills is affected by the relative supply of those assets.

[14] See footnote 10.

[15] The default controls, which Krishnamurthy and Vissing-Jorgensen include in their regressions explaining that spread, are statistically not different from zero.

Table 1: Selected emprical moments

$corr\left(s_t^{Treas}, bills_t/gdp_t\right)$	-0.62
$corr\left(R_t, R_t^m\right)$	0.99
$corr\left(s_t^{Euler}, R_t^m\right)$	-0.82
$corr\left(R_t^{Euler}, R_t^m\right)$	0.53
$sd(s_t^{Euler})/sd(R_t^m)$	0.85
$sd(s_t^{Euler})/sd(R_t^{Euler})$	1.49

Note: Standard deviations refer to net interest rates.

the T-bills held by the private sector $(bills_t)$. The first line of Table 1 displays the correlation between the short-term spread s_t^{Treas}, identified as the spread between the 3-month high-grade commercial paper rate and the 3-month T-bill rate, and the bills-to-GDP ratio, where we used the total amount of outstanding T-bills minus the amount of T-bills held by the Federal Reserve.[16] The empirical correlation is strongly negative (-0.62), indicating that the supply of eligible assets (T-bills) matters for the spread.[17] For the sake of completeness, the second line gives the correlation (0.99) between the T-bills rate R_t and the Federal Funds rate R_t^m, which is well-known to be close to unity (see e.g. Sarno and Thornton, 2003). Given that the Fed controls the Federal Funds rate and influences the private sector holdings of T-bills, these results suggest that monetary policy plays a crucial role for the liquidity premium on short-term treasuries.

2.2   Euler equation rate

As demonstrated by Canzoneri et al. (2007), Atkeson and Kehoe (2009), and Chari et al. (2009), the rate implied by the consumption Euler equation – henceforth, the "Euler equation rate" – hardly mimics the US monetary policy rate, i.e. the Federal Funds rate. Applying different approaches, Canzoneri et al. (2007) and Atkeson and Kehoe (2009) both find that the spread between the Euler equation rate and the Federal Funds rate is actually negatively related to the Federal Funds rate. Chari et al. (2009) further show that this spread, which they call the "Euler equation error", is more than six times larger than the short-term interest rate, which they view as one of several critical properties of standard (New Keynesian) models.

To assess our model's ability to explain this pattern, we follow Canzoneri et al. (2007) and compute the empirical interest rate implied by standard consumption Euler equations. According

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[16] The sample covers 2003-2007, with quarterly data, due to data availability of privately held T-bills and to remove the recent financial crisis episode. The sample can be extended back to 1970 if total outstanding T-bills – instead of privately held T-bills – are considered. For the full sample (1970-2007), the correlation is less pronounced and equals -0.36. Data are from the Federal Reserve Bank of St. Louis FRED database, the U.S. Treasury, and the Federal Reserve Board.

[17] We find a similar correlation with long-term spreads, as with estimates presented by Krishnamurthy and Vissing-Jorgensen (2012). When supply is measured as the total government debt outstanding relative to GDP, and the spread between yields on AAA corporate bonds and long-term government bonds is considered, the correlation is -0.62. The sample covers 1934-2007, with yearly data. The debt-to-GDP ratio data is from Bohn (2008).

to the consumption Euler equation, the (gross) Euler equation rate R_t^{Euler} satisfies $1/R_t^{Euler} = \beta E_t \left[(u_{c,t+1}/u_{c,t})(P_t/P_{t+1}) \right]$, where β is the discount factor, $u_{c,t}$ is marginal utility of consumption, and P_t is the aggregate price level. With a standard CRRA utility function, leading to a marginal utility of consumption $u_{c,t} = c_t^{-\sigma}$, and under conditional log-normality the consumption Euler equation can be written as

$$\frac{1}{R_t^{Euler}} = \beta \exp \left[\begin{array}{c} -\sigma \left(E_t \log c_{t+1} - \log c_t \right) - E_t \log \pi_{t+1} \\ +\frac{\sigma^2}{2} var_t \log c_{t+1} + \frac{1}{2} var_t \log \pi_{t+1} + \sigma cov_t \left(\log c_{t+1}, \log \pi_{t+1} \right) \end{array} \right], \qquad (1)$$

where $\pi_t = P_t/P_{t-1}$. Equation (1) is used to compute the implied Euler equation rate R_t^{Euler}, where the conditional moments are estimated from a six-variable VAR, $Y_t = A_0 + A_1 Y_{t-1} + v_t$, assuming $v \sim i.i.d.N(0, \Sigma)$, $\sigma = 1$ and $\beta = .993$.[18] These parameters are chosen according to our calibration strategy (see Section 5.1). The variables included in Y_t (1966Q1-2007Q4) are log per capita real personal consumption expenditures on nondurable goods and services, log change in the deflator of this consumption measure, log price of industrial commodities, log per capita real disposable personal income, Federal Funds rate, and log per capita real non-consumption GDP.[19] Moreover, a segmented (1974Q1) time trend is included in A_0.

The Federal Funds rate and the Euler equation rate,[20] which should be identical according to standard macroeconomic models, are positively correlated by 0.53 (see Table 1). The spread between the computed standard Euler interest rate and Federal Funds rate, $s_t^{Euler} = R_t^{Euler} - R_t^m$ is strongly negatively correlated with the Federal Funds rate (-0.82), which accords to Atkeson and Kehoe's (2008) result for the Euler equation error using Smets and Wouters' (2007) model. The standard deviation of the spread between the Euler equation rate and the policy rate is 2.78, compared to 1.87 for the Euler equation rate and 3.26 for the Federal Funds rate. Hence, the spread is less volatile than the Federal Funds rate and is much more volatile than the Euler equation rate; the latter relation being less pronounced than for Smets and Wouters' (2007) model, according to Chari et al. (2009).

We further examine the effect of a shock to the policy rate on the Euler equation rate implied by the response of consumption and inflation using the VAR described above, and on the spread between the Euler rate and the policy rate. The ordering is the price of industrial commodities, personal income, non-consumption GDP, consumption expenditures on nondurable goods and services, change in the deflator of that consumption measure, and the Federal Funds rate.[21] Figure

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[18] Our results differ slightly from Canzoneri et al. (2007), who use a CRRA utility function with $\sigma = 2$.

[19] Data are from the Federal Reserve Bank of St. Louis FRED database and are released by the Federal Reserve Board, the Bureau of Economic Analysis (U.S. Department of Commerce), the Bureau of Labor Statistics (U.S. Department of Labor), and the Census Bureau (U.S. Department of Commerce).

[20] The computed Euler rate, the Federal Funds rate, and the spread between these two rates, $s_t^{Euler} = R_t^{Euler} - R_t^m$, are displayed in Appendix B.

[21] Similar impulse response functions are obtained with the ordering of Fuhrer (2000) and Canzoneri et al. (2007), or when the generalized impulses decomposition method is used.
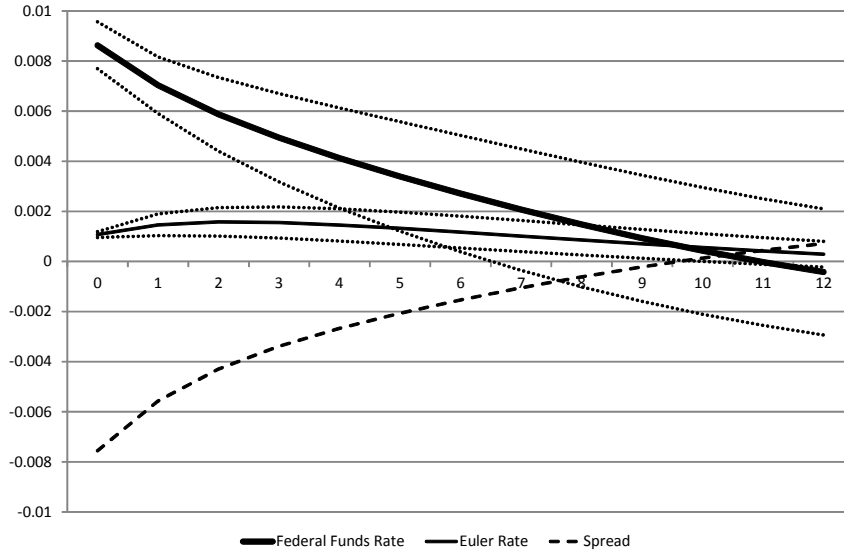
Figure 3: Impulse response functions for a Federal funds rate shock

3 contains the responses of the implied Euler equation rate, the Federal Funds rate, and the spread between the two, to a Federal Funds rate shock. The dotted lines display 95% confidence intervals for the impulse responses. Following a positive Federal funds rate shock, the nominal Euler equation rate increases as well but by only one-fifth of the increase in the funds rate. This implies a decline in the spread between the two rates that lasts about two years.

To summarize, the spread between the Euler equation rate and the observed policy rate, which are equated in standard models, exhibits two properties, a systematic negative response to policy rate changes and a high relative volatility. Both properties cast doubts on the validity and the precision of the transmission mechanism of standard macroeconomic models.[22] The following analysis will show that these observations can be explained by a liquidity premium on short-term treasuries. Specifically, instead of considering exogenous residuals to address the discrepancy between the policy rate and the marginal rate of intertemporal substitution, our framework explains the wedge between these two variables with the endogenous liquidity premium which originates from monetary policy implementation. In the subsequent analysis, we show that a small modification of a standard sticky price model, i.e., the explicit specification of open market operations, leads to a dynamic behavior of a liquidity premium that matches all observations presented in this section; the latter being entirely unexplained by standard macroeconomic models.

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[22] This seems to be independent of the inclusion of consumption habits (see Canzoneri et al. 2007, Atkeson and Kehoe, 2009, and Chari et al., 2009).

3 The model

In this section, we develop a macroeconomic framework which can explain the results presented in the previous section. The model is sufficiently stylized to facilitate comparisons with standard macroeconomic models and to be able to derive analytical results (for specific parameter values). For simplicity, we abstract from financial intermediation and assume that households directly trade with the central bank in open market operations. They hold short-term treasuries (i.e. one-period government bonds), equity, and non-interest bearing money. Their demand for money is induced by assuming that goods market transactions cannot be conducted by using credit. To allow for a transparent specification of markets and the timing of events, we apply a basic cash-in-advance constraint (which is replaced by a money-in-the-utility-function specification in Section 5.4). We consider that the central bank supplies money in exchange for eligible securities. Specifically, we restrict our attention to the case where only short-term treasuries are eligible in open market operations, which accords to the common practice of central banks (like the BoE or the US-Fed before 2007) of restricting the set of eligible securities mainly to short-term government debt. Open market operations are conducted in form of outright sales/purchases and repurchase agreements, which allows to replicate the empirically relevant case where money is held by the private sector and is typically not withdrawn by the central bank.[23] To be able to calibrate the model in an appropriate way, we further consider capital accumulation, which is, however, not essential for the main results of the paper.

Throughout the paper, upper case letters denote nominal variables and lower case letters real variables. Though, agents are not heterogenous, we introduce indices for individual agents (i, j, and k) to describe individual choices in a transparent way.

3.1 Timing of events

There are infinitely many households, firms, and retailer indexed with $i \in [0, 1]$, $j \in [0, 1]$, and $h \in [0, 1]$. A household i enters a period t with assets carried over from the previous period $t - 1 : M_{i,t-1}^H + B_{i,t-1} + V_t z_{i,t-1}$, where $M_{i,t}^H \geq 0$ denotes holdings of money, $B_{i,t} \geq 0$ one-period nominally risk-free government bonds, and $z_{i,t-1} \in [0, 1]$ shares of firms valued at the price V_t. At the beginning of the period, aggregate shocks materialize, labor is supplied by households, intermediate goods are produced by firms and sold to retailers.

Then, households and the central bank participate in open market operations, where money is supplied outright or under repurchase agreements (repos) against eligible securities. The relative

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[23] The central bank thus creates a "structural deficiency" when it supplies money, by choosing a suited relation between money supplied outright and under repurchase agreements. This strategy has for example been applied by the US-Federal Reserve: "*To most effectively influence the level of reserve balances, the Federal Reserve has created what is called a 'structural deficiency'. That is, it has created permanent additions to the supply of reserve balances that are somewhat less than the total need. Then on a seasonal and daily basis, the Desk is in a position to add balances temporarily to get to the desired level.*" (see "Fedpoint: Open Market Operations", http://www.newyorkfed.org/aboutthefed/fedpoint/fed32.html).

price of money R_t^m is controlled by the central bank and will be called policy rate. Assuming that only government bonds are eligible, household i faces the following constraint:

$$I_{i,t} \leq B_{i,t-1}/R_t^m, \tag{2}$$

which we summarize as the "collateral constraint".[24] Its bond holdings then equals $B_{i,t-1} - \Delta B_{i,t}^C$, where $\Delta B_{i,t}^C$ denotes treasuries received by the central bank.

In the final goods market, money is assumed to be the only accepted means of payment for a household i, who buys consumption goods $c_{i,t}$, as well as for a firm j, who buys investment goods $x_{j,t}$. Before the goods market opens, the latter borrows money $L_{j,t}$ from households at the price $1/R_t^L$, such that it is constrained by $L_{j,t}/R_t^L \geq P_t x_{j,t}$, where P_t denotes the price level, while household $i's$ goods market expenditures are restricted by

$$P_t c_{i,t} \leq I_{i,t} + M_{i,t-1}^H - L_{i,t}/R_t^L. \tag{3}$$

where $L_{i,t} = \int L_{i,j,t} dj$. Before the asset market opens, it can repurchase treasuries, $B_{i,t}^R = R_t^m M_{i,t}^R$, such that its bond holdings equals $B_{i,t-1} - \Delta B_{i,t}^C + B_{i,t}^R$. In the asset market, loans are repaid and households trade money, equity, and treasuries at the price $1/R_t$ subject to

$$(B_{i,t}/R_t) + M_{i,t}^H + V_t z_{i,t} - (V_t + P_t \varrho_t) z_{i,t-1} + R_t^m M_{i,t}^R + P_t c_{i,t} + P_t \tau_t \tag{4}$$
$$\leq B_{i,t-1} - \Delta B_{i,t}^C + B_{i,t}^R + I_{i,t} + M_{i,t-1}^H + L_{i,t} \left(1 - 1/R_t^L\right) + P_t w_t n_{i,t} + P_t \varphi_t,$$

where w_t denotes the real wage rate, $n_{i,t}$ working time, τ_t a lump-sum tax, ϱ_t dividends from intermediate goods producing firms, and φ_t profits from retailer. The central bank reinvests its payoffs from maturing government bonds in newly issued bonds and leaves aggregate money supply unchanged at this stage, $\int_0^1 M_{i,t}^H di = \int_0^1 (M_{i,t-1}^H + I_{i,t} - M_{i,t}^R) di$.

Note that we abstract from an intraperiod money market (like an overnight interbank market), which would allow exchanging federal funds among according to idiosyncratic liquidity demands within the maintenance period. Since the price of federal funds charged in an intraperiod money market would – without further imperfections – be equal to the price charged by the central bank, we identify the central bank target rate with the price of money in open market operations R_t^m.

3.2 Private sector

Households Households are infinitely lived and have identical endowments and preferences. Household i maximizes the expected sum of a discounted stream of instantaneous utilities u:

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{i,t}, n_{i,t}\right), \tag{5}$$

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[24] We refer to the fact that repos are essentially collateralized loans, while we apply the term "collateral" in a broader sense also for outright operations.

where E_0 is the expectation operator conditional on the time 0 information set, and $\beta \in (0,1)$ is the subjective discount factor. The instantaneous utility function accounts for external habits and is assumed to satisfy $u\left(c_{i,t}, n_{i,t}\right) = [(c_{i,t} - h \cdot c_{t-1})^{1-\sigma}/(1-\sigma)^{-1}] - \theta n_{i,t}^{1+\sigma_n}/(1+\sigma_n)$, where $\sigma \geq 1$, $\sigma_n \geq 0$, $\theta > 0$, and $h \geq 0$.

As described above, household i faces three constraints in each period. In open market operations, it can acquire additional money $I_{i,t}$ up to the amount of government bonds carried over from the previous period $B_{i,t-1}$ discounted by R_t^m (see 2). Hence, other assets (e.g. equity) are not eligible in open market operations. Throughout the analysis, we restrict our attention to the case where money is not withdrawn from the private sector ($I_{i,t} < 0$), which requires a sufficiently large fraction of money supplied under repurchase agreements (see Section 3.3). Households are further assumed to rely on cash for transactions in the goods market (see 3). In the asset market, household i receives payoffs from maturing bonds and dividends. It can buy bonds from the government and invest in shares of intermediate goods producing firms $j \in [0,1]$, $z_{i,t} = \int z_{i,t}^j dj$. Using $B_{i,t}^R = R_t^m M_{i,t}^R$ and $\Delta B_{i,t}^c = R_t^m I_{i,t}$, household $i's$ asset market constraint (4) can be rewritten as

$$
(B_{i,t}/R_t) + M_{i,t}^H + (R_t^m - 1) I_{i,t} + V_t z_{i,t} + P_t c_{i,t} + P_t \tau_t \tag{6}
$$
$$
\leq B_{i,t-1} + M_{i,t-1}^H + (V_t + P_t \varrho_t) z_{i,t-1} + L_{i,t} \left(1 - 1/R_t^L\right) + P_t w_t n_{i,t} + P_t \varphi_t,
$$

while its $i's$ borrowing is restricted by $z_{i,t} \geq 0$, by $M_{i,t}^H \geq 0$, and $B_{i,t} \geq 0$ $\forall t \geq 0$. The term $(R_t^m - 1) I_{i,t}$ measures the costs of money acquired in open market operations. Maximizing (5) subject to (2), (3), (6) and the borrowing constraints, for given initial values $M_{i,-1}^H$, $B_{i,-1}$, and $z_{i,-1}$ leads to the following first order conditions for working time, consumption, $-u_{i,nt}/w_t = \lambda_{i,t}$ and $u_{i,ct} = \lambda_{i,t} + \psi_{i,t}$, as well as for additional money, holdings of government bonds, money, equity, and intraperiod loans

$$
R_t^m \left(\lambda_{i,t} + \eta_{i,t}\right) = \lambda_{i,t} + \psi_{i,t}, \tag{7}
$$
$$
\beta E_t \left[\left(\lambda_{i,t+1} + \eta_{i,t+1}\right) \pi_{t+1}^{-1}\right] = \lambda_{i,t}/R_t, \tag{8}
$$
$$
\beta E_t \left[\left(\lambda_{i,t+1} + \psi_{i,t+1}\right) \pi_{t+1}^{-1}\right] = \lambda_{i,t}, \tag{9}
$$
$$
\beta E_t \left[\lambda_{i,t+1} R_{t+1}^q \pi_{t+1}^{-1}\right] = \lambda_{i,t}, \tag{10}
$$
$$
\psi_{i,t} + \lambda_{i,t} = R_t^L \lambda_{i,t}. \tag{11}
$$

where $R_t^q = (V_t + P_t \varrho_t)/V_{t-1}$ is the nominal rate of return on equity, and $\eta_{i,t}$, $\psi_{i,t}$, and $\lambda_{i,t}$ denote the multiplier on the collateral constraint (2), the goods market constraint (3), and the asset market constraint (6). Finally, the following complementary slackness conditions hold in the household's optimum i.) $0 \leq b_{i,t-1} \pi_t^{-1} - R_t^m i_{i,t}$, $\eta_{i,t} \geq 0$, $\eta_{i,t} \left(b_{i,t-1} \pi_t^{-1} - R_t^m i_{i,t}\right) = 0$, and ii.) $0 \leq i_{i,t} + m_{i,t-1}^H \pi_t^{-1} - c_{i,t}$, $\psi_{i,t} \geq 0$, $\psi_{i,t} \left(i_{i,t} + m_{i,t-1}^H \pi_t^{-1} - c_{i,t}\right) = 0$, where $m_{i,t}^H = M_{i,t}^H/P_t$, $b_{i,t} = B_{i,t}/P_t$, and $i_{i,t} = I_{i,t}/P_t$, as well as (6) with equality and associated transversality conditions.

The first order conditions for working time and consumption show that $\lambda_{i,t} > 0$ and that a

10

binding goods market constraint, $\psi_{i,t} > 0$, distorts the intratemporal consumption-leisure decision in a conventional way, $u_{i,ct} = -(u_{i,nt}/w_t) + \psi_{i,t}$. Condition (8) shows that a rise in the multiplier $\eta_{i,t}$, which measures the liquidity value of treasuries, tends to lower the treasury rate R_t. Hence, a positive liquidity value of treasuries $\eta_{i,t} > 0$ gives rise to a liquidity premium between the treasury rate and the rate of return on equity, as can be seen from combining (8) and (10): $R_t E_t \left[\left(\lambda_{i,t+1} + \eta_{i,t+1} \right) \pi_{t+1}^{-1} \right] = E_t \left[R_{t+1}^q \cdot \lambda_{i,t+1} \pi_{t+1}^{-1} \right]$. The household's investment decisions further relate the treasury rate to the policy rate. It is willing to hold both assets, money and treasuries, if the rate of return on treasuries compensates for the costs of acquiring new money in the next period. This can be seen by combining $u_{i,ct} = \lambda_{i,t} + \psi_{i,t}$, (7), (8), and (9) to

$$1/R_t = \frac{E_t \left[\left(1/R_{t+1}^m \right) \left(u_{i,ct+1}/\pi_{t+1} \right) \right]}{E_t \left[\left(u_{i,ct+1}/\pi_{t+1} \right) \right]}, \tag{12}$$

implying that the interest rate on treasuries equals the expected future policy rate up to first order in accordance with evidence from the US treasury yield curve (see e.g. Simon 1990, and Lange et al., 2003). Eliminating the multiplier in condition (11) with $u_{i,ct} = \lambda_{i,t} + \psi_{i,t}$ and (9), shows that the interest rate on intraperiod loans R_t^L equals $R_t^L = 1/\{\beta E_t \left[(u_{i,ct+1}/u_{i,ct})\pi_{t+1}^{-1} \right]\}$.

Firms   There are perfectly competitive intermediate goods producing firms, which sell their goods to monopolistically competitive retailers. The latter sell a differentiated good to bundlers who assemble final goods using a Dixit-Stiglitz technology.

There is a continuum of intermediate goods producing firms indexed with $j \in [0,1]$. They are perfectly competitive, owned by the households, and produce an identical intermediate good with labor $n_{j,t}$ and physical capital $k_{j,t-1}$. Firm j produces according to the production function

$$y_{j,t}^{im} = a_t n_{j,t}^\alpha k_{j,t-1}^{1-\alpha}, \;\; \alpha \in (0,1), \tag{13}$$

and sells the intermediate good to retailers at the price P_t^m. The productivity level a_t is generated by a stochastic process satisfying $a_t = a_{t-1}^{\rho_a} \exp \varepsilon_{a,t}$, where $\rho_a \geq 0$, and $\varepsilon_{a,t}'s$ are normally and i.i.d. distributed with $E_{t-1}\varepsilon_{a,t} = 0$ and a constant standard deviation $st.dev.(\varepsilon_a) \geq 0$. The accumulation of physical capital is associated with adjustment costs: $k_{j,t} = (1-\delta) k_{j,t-1} + x_{j,t}G_{j,t}$, where $x_{j,t}$ denotes investment expenditures and investment adjustment costs are $G\left(x_{j,t}/x_{j,t-1} \right)$ $= 1 - \xi\frac{1}{2}\left(x_{j,t}/x_{j,t-1} - 1 \right)^2$ with $G\left(1 \right) = G'\left(1 \right) = 0$ and $G''(1) = \xi > 0$. Intermediate goods producing firms also rely on money for goods market purchases, which they borrow from households at the price $1/R_t^L$ and pay back after goods are sold, such that loan demand satisfies

$$L_{j,t}/R_t^L \geq P_t x_{j,t}. \tag{14}$$

The problem of a firm j can then be summarized as $\max E_t \sum_{k=0}^\infty p_{t,t+k}\varrho_{j,t+k}$, where $p_{t,t+k} = \beta^k \lambda_{i,t+k}/\lambda_{i,t}$ denotes the stochastic discount factor and $\varrho_{j,t}$ real dividends $\varrho_{j,t} = (P_t^m/P_t)a_t n_{j,t}^\alpha k_{j,t-1}^{1-\alpha} - w_t n_{j,t} - x_{j,t} - (1 - 1/R_t^L)L_{j,t}/P_t$, subject to capital accumulation and the cash constraint (14).

The first order conditions for working time, loans, investments, and capital can be summarized by $w_t = mc_t a_t \alpha y_{j,t}^{im} n_{j,t}^{-1}$,

$$R_t^L = q_t \left[G_{j,t} + (x_{j,t}/x_{j,t-1}) G'_{j,t} \right] - E_t \left[p_{t,t+1} q_{t+1} (x_{j,t+1}/x_{j,t})^2 G'_{j,t+1} \right], \tag{15}$$

$$q_t = E_t \left[p_{t,t+1} \left((1-\alpha) mc_{j,t+1} y_{j,t+1}^{im} k_{j,t}^{-1} + (1-\delta) q_{t+1} \right) \right], \tag{16}$$

where $mc_t = P_t^m/P_t$ denotes real marginal costs and q_t the price of physical capital in terms of the final good. Condition (15) shows that the investment decision is distorted by the loan rate R_t^L, which originates from the cash constraint (14). Given that intermediate goods producing firms face the same prices, they will behave in an identical way.

Monopolistically competitive retailers buy intermediate goods $y_t^{im} = \int_0^1 y_{j,t}^{im} dj$ at the common price P_t^m. A retailer $h \in [0,1]$ relabels the intermediate good to $y_{h,t}$ and sells it at the price $P_{h,t}$ to perfectly competitive bundlers, who bundle the goods $y_{h,t}$ to the final consumption good y_t with the technology, $y_t^{\frac{\varepsilon-1}{\varepsilon}} = \int_0^1 y_{h,t}^{\frac{\varepsilon-1}{\varepsilon}} dh$, where $\varepsilon > 1$. The cost minimizing demand for $y_{h,t}$ is therefore given by $y_{h,t} = (P_{h,t}/P_t)^{-\varepsilon} y_t$. Retailers set their prices to maximize profits, where we consider a nominal rigidity in form of staggered price setting. Each period a measure $1 - \phi$ of randomly selected retailers may reset their prices independently of the time elapsed since the last price setting, while a fraction $\phi \in [0,1)$ of retailers do not adjust their prices. The fraction $1 - \phi$ of retailers set their price to maximize the expected sum of discounted future profits, $\max_{\overline{P}_{h,t}} E_t \sum_{s=0}^{\infty} \phi^s \varphi_{t,t+s} (\overline{P}_{h,t} y_{h,t+s} - P_{t+s} mc_{t+s} y_{h,t+s})$, s.t. $y_{h,t+s} = \overline{P} P_{h,t}^{-\epsilon} P_{t+s}^{\epsilon} y_{t+s}$, where real marginal costs are given by $mc_t = P_t^m/P_t$. The first order condition for their price $\overline{P}_{h,t}$ is given by $Z_t = \frac{\varepsilon}{\varepsilon-1} Z_{1,t}/Z_{2,t}$, where $Z_t = \overline{P}_{h,t}/P_t$, $Z_{1,t} = c_t^{-\sigma} y_t mc_t + \phi \beta E_t \pi_{t+1}^{\varepsilon} Z_{1,t+1}$ and $Z_{1,t} = c_t^{-\sigma} y_t + \phi \beta E_t \pi_{t+1}^{\varepsilon-1} Z_{2,t+1}$. With perfectly competitive bundlers and the homogenous bundling technology, the price index P_t for the final consumption good satisfies $P_t^{1-\varepsilon} = \int_0^1 P_{h,t}^{1-\varepsilon} dh$. Using the demand constraint, we obtain $1 = (1-\phi) Z_t^{1-\varepsilon} + \phi \pi_t^{\varepsilon-1}$.

Aggregate intermediate output is then given by $y_t^{im} = a_t n_t^{\alpha} k_{t-1}^{1-\alpha}$, where $n_t = \int_0^1 n_{j,t} dj$, while price dispersion across retailers affects aggregate output. Specifically, the market clearing condition in the intermediate goods market, $y_t^{im} = \int_0^1 y_{h,t} dh$, gives $a_t n_t^{\alpha} k_{t-1}^{1-\alpha} = \int_0^1 (P_{h,t}/P_t)^{-\varepsilon} y_t dh \Leftrightarrow y_t = a_t n_t^{\alpha} k_{t-1}^{1-\alpha}/s_t$, where $s_t \equiv \int_0^1 (P_{h,t}/P_t)^{-\varepsilon} dh$ and $s_t = (1-\phi) Z_t^{-\varepsilon} + \phi s_{t-1} \pi_t^{\varepsilon}$ given s_{-1}.

3.3 Public sector

Fiscal authority The government issues one-period nominally risk-free bonds B_t^T, which are either held by households B_t^H or the central bank B_t^C. Throughout, we also refer to these bonds as *T-bills* to emphasize that B_t^T consists of short-term treasuries that typically serve as collateral in open market operations. To minimize interactions between fiscal policy and monetary policies, which are beyond the scope of the analysis, we assume that the supply of government bonds is exogenously determined (like in Lagos, 2010, or Shi, 2012). Specifically, we consider a simple bond

supply regime that keeps the growth rate of T-bills constant,

$$B_t^T = \Gamma B_{t-1}^T, \tag{17}$$

where $\Gamma > \beta$. As mentioned above, (17) describes the supply of treasury securities that are declared as eligible by the central bank, and is not aimed at modelling the evolution of total public debt. The latter usually also contains debt with longer maturity that might grow with a rate different from Γ, which will not be modelled here to keep the exposition simple.0 In order to avoid any further effects of fiscal policy, we assume that the government can raise tax revenues in a non-distortionary way, $P_t \tau_t$. Accounting for transfers $P_t \tau_t^m$ from the central bank, the government budget constraint is given by $\left(B_t^T / R_t\right) + P_t \tau_t^m + P_t \tau_t = B_{t-1}^T$.

Central bank The central bank supplies money in exchange for treasuries in open market operations in form of outright sales/purchases M_t^H and repurchase agreements M_t^R. At the beginning of each period, the central bank's stock of treasuries equals B_{t-1}^C and the stock of outstanding money equals M_{t-1}^H. It then receives an amount ΔB_t^C of treasuries in exchange for newly supplied money $I_t = M_t^H - M_{t-1}^H + M_t^R$, and after repurchase agreements are settled its holdings of treasuries and the amount of outstanding money reduce by B_t^R and by $M_t^R = B_t^R$, respectively. Before the asset market opens, where the central bank can invest in new T-bills B_t^C, it holds an amount equal to $\Delta B_t^C + B_{t-1}^C - B_t^R$. Its budget constraint is thus given by $\left(B_t^C / R_t\right) + P_t \tau_t^m = \Delta B_t^C + B_{t-1}^C - B_t^R + M_t^H - M_{t-1}^H - \left(I_t - M_t^R\right)$. Substituting out I_t, B_t^R, and ΔB_t^C using $\Delta B_t^C = R_t^m I_t$, it can be simplified to $\left(B_t^C / R_t\right) - B_{t-1}^C = R_t^m \left(M_t^H - M_{t-1}^H\right) + \left(R_t^m - 1\right) M_t^R - P_t \tau_t^m$. Following common central bank practice, we assume that interest earnings are transferred to the government, $P_t \tau_t^m = B_t^C \left(1 - 1/R_t\right) + \left(R_t^m - 1\right) \left(M_t^H - M_{t-1}^H + M_t^R\right)$, and that maturing assets are rolled over (see e.g. Meulendyke, 1998, Ch.7).[25] Accordingly, the central bank holdings of treasuries will evolve according to $B_t^C - B_{t-1}^C = M_t^H - M_{t-1}^H$. Further restricting initial values to $B_{-1}^C = M_{-1}^H$, leads to the central bank balance sheet constraint

$$B_t^C = M_t^H. \tag{18}$$

Regarding the implementation of monetary policy, we assume that the central bank conducts monetary policy by using a standard feedback rule for the current policy rate R_t^m and by choosing an average policy rate $R^m > 1$:

$$R_t^m = \left(R_{t-1}^m\right)^{\rho_R} \left(R^m\right)^{1-\rho_R} \left(\pi_t / \pi\right)^{\rho_\pi (1-\rho_R)} [(y_t/y)/(\widetilde{y}_t/\widetilde{y})]^{\rho_y (1-\rho_R)} \exp \varepsilon_{r,t}, \tag{19}$$

---

[25] Note that central bank transfers cannot be negative in equilibrium, such that the central bank will never rely on funds from the government. A discussion of the role of government transfers for central bank independence can be found in Sims (2003).

where $R^m > 1$, $\rho_R \geq 0$, $\rho_\pi \geq 0$, and $\rho_y \geq 0$, $\widetilde{y}_t$ denotes first-best output, which is defined in Appendix A.1, and the $\varepsilon'_{r,t}s$ are normally and i.i.d. with $E_{t-1}\varepsilon_{r,t} = 0$. The central bank further sets an inflation target, which is consistent with the long-run inflation rate and satisfies $\pi > \beta$. To avoid further complexities, we will assume that the growth rate of T-bills Γ equals the central bank's inflation target, $\Gamma = \pi$, which actually accords to the estimated growth rate of T-bills (corrected by GDP growth) for the sample period 1966-2007 (see Section 5.1). The model can, however, easily be extended to allow for $\Gamma \neq \pi$ (see Schabert, 2013).[26]

The central bank can further decide on whether money is traded in form of outright sales/purchases or in form of repurchase agreements. For simplicity, we assume that it exogenously sets the ratio of money supplies under both types of open market operations $\Omega : M_t^R = \Omega \cdot M_t^H$. We assume that $\Omega > 0$ in accordance with the practice of the US Fed, i.e. with the fact that the Trading Desk of the New York Fed "structures its outright holdings to maintain a need to routinely add to balances by arranging repurchase agreements" (see Federal Reserve Bank of New York, 2006).[27]

3.4 Rational expectations equilibrium

In equilibrium, there will be no arbitrage opportunities and markets clear, $n_t = \int_0^1 n_{j,t}dj = \int_0^1 n_{i,t}di$ and $y_t = c_t + x_t$, where $x_t = \int_0^1 x_{j,t}dj$ and $c_t = \int_0^1 c_{i,t}di$. Aggregate asset holdings satisfy $\forall t \geq 0$: $\int z_{i,t}di = 1$, $B_t = \int B_{i,t}di$, $M_t^R = \int_0^1 M_{i,t}^R di$, $M_t^H = \int_0^1 M_{i,t}^H di$, $\int I_{i,t}di = I_t = M_t^H - M_{t-1}^H + M_t^R$, $L_t = \int L_{j,t}dj = \int L_{i,t}di$, and $B_t^T = B_t + B_t^C$. Given that households (firms) behave in an identical way, we will omit indices in the subsequent analysis. In a rational expectations (REE) all plans and constraints of households and firms are satisfied and consistent with monetary and fiscal policy, for given initial endowments (see Definition 1 in Appendix A.1).

The main difference to a standard New Keynesian model is the existence of the collateral constraint in open market operations (2), which restricts households' access to money. The model reduces to a conventional sticky price model if the collateral constraint,

$$M_t^H - M_{t-1}^H + M_t^R \leq B_{t-1}/R_t^m, \tag{20}$$

is slack, i.e. if the multiplier η_t equals zero. In this case, there is no liquidity premium on eligible securities, such that the expected equity return equals the treasury rate up to first order (see 8 and 10). Throughout the subsequent analysis, we are particularly interested in the case where

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[26] If the central bank would adjust the amount of eligible treasuries, it can chose an inflation target that differs from Γ. When, for example, the central bank chooses for a smaller inflation target $\pi < \Gamma$, it might accept smaller fractions of treasuries in open market operations. Otherwise, for $\pi > \Gamma$, it might also declare other assets (or a fraction of them) as eligible, which grow with a rate that exceeds Γ.

[27] The choice of Ω does not affect the pattern but only the size of monetary policy effects, i.e. the size of the responses to an innovation to the policy rate, which will be taken into account for the calibration of the model. A higher ratio of money supplied under repurchase agreements relative to money supplied outright actually increases the effectiveness of changes in the policy rate, which provides a rationale why central banks – like the US Fed – create a structural deficiency with respect to the outright supply of money.

the collateral constraint (20) is binding. To see when this is the case, eliminate $\lambda_{i,t}$ and $\psi_{i,t}$ by $u_{i,ct} = \lambda_{i,t} + \psi_{i,t}$ in (7) and in (9), which leads to $\frac{\eta_{i,t}}{u_{i,ct}} = \frac{1}{R_t^m} - \beta E_t \frac{u_{i,ct+1}}{u_{i,ct}\pi_{t+1}}$. In equilibrium, it can be written as

$$\eta_t / u_{c,t} = (1/R_t^m) - (1/R_t^{Euler}) \geq 0, \tag{21}$$

where R_t^{Euler} is the consumption Euler equation rate defined as $1/R_t^{Euler} = \beta E_t \frac{u_{c,t+1}}{u_{c,t}\pi_{t+1}}$ (as in Section 2). As it is well-known, the nominal Euler equation rate measures the equilibrium valuation of money. Agents are willing to spend $R_t^{Euler} - 1$ to transform one unit of an illiquid asset, i.e. an asset that is not accepted as a means of payment today and delivers one unit of money tomorrow, into one unit of money today. Hence, if the central bank supplies money at a lower price $R_t^m < R_t^{Euler}$, households earn a positive rent and are willing to get the maximum amount of money. Given that this amount is restricted by holdings of eligible assets, the collateral constraint (20) will be binding, indicating a positive liquidity value of treasuries, $\eta_t > 0$. A binding collateral constraint (20), which relies on a positive valuation of liquidity, implies the cash constraint (3) to be binding as well, $\psi_t > 0$.[28]

If the central bank would supply money at the rate R_t^m in an unrestricted way (e.g. by accepting securities that can be issued by private sector agents in an unbounded way), then households will adjust their consumption sequence until their marginal valuation of money equals the price, i.e. $R_t^{Euler} = R_t^m$. This accords to the case typically considered in standard macroeconomic models, where the policy rate is identical to the marginal rate of intertemporal substitution and there is no liquidity premium on treasuries (see Definition 2 in Appendix A.1).

4 Analytical results

In this Section, we analytically examine some main properties of the model. In particular, we show that the liquidity premium is affected by the bills-to-gdp ratio and by monetary policy and is more volatile than the Euler equation rate, in accordance with the empirical evidence provided in Section 2. For this, we apply several parameter values that simplify the model. Specifically, we assume that capital is not productive ($\alpha = 1$), utility is not characterized by habits ($h = 0$) and logarithmic in consumption ($\sigma = 1$), the inflation target equals one ($\pi = 1$), and that money is only supplied via repos ($\Omega \to \infty$). The latter assumption implies that T-bills are solely held by the private sector, $B_t = B_t^T$.

Euler equation rate versus policy rate We first examine how the spread between the Euler equation rate R_t^{Euler} and the policy rate R_t^m is related to monetary policy and the bills-to-gdp ratio. Throughout the following analysis, we restrict our attention to the case where the central bank sets the policy rate below the equilibrium Euler equation rate. As implied by (21), the

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[28]To see this, combine household $i's$ first order conditions $u_{i,ct} = \lambda_{i,t} + \psi_{i,t}$ and (9), which lead to $c_{i,t}^{-\sigma} = \beta E_t(c_{i,t+1}^{-\sigma}/\pi_{t+1}) + \psi_{i,t}$. Hence, the multiplier ψ_t on the goods market constraint (3) satisfies $\psi_t/u_{c,t} = 1 - (1/R_t^{Euler}) \geq 0$ in equilibrium, which obviously implies $\psi_t > 0$ if $1 \leq R_t^m < R_t^{Euler}$.

collateral constraint (20) is binding in this case. For perfectly flexible prices, $\phi = 0$, it can be shown that this can be guaranteed by the central bank if it sets the policy rate in a way that its expected value $E_t R_{t+1}^m$ is below the long-run Euler equation rate, which equals π/β as usual. The spread $s_t^{Euler} = R_t^{Euler} - R_t^m$, which proxies the liquidity premium on treasuries $(R_t^{Euler} - R_t)$ as the treasury rate closely follows the expected policy rate (see 12), can further be shown to be negatively related the expected policy rate and to the expected bills-to-gdp ratio. These properties are summarized in the following proposition.

Proposition 1 *Consider a simplified model version with $\phi = 0$, $h = 0$, $\alpha = 1$, $\Omega \to \infty$, $\sigma = 1$, and $\Gamma = 1$. If the central bank sets the policy rate and the inflation target according to $E_t R_{t+1}^m < \pi/\beta$ and $\pi > \beta$, the collateral constraint (20) is binding in equilibrium. The spread s_t^{Euler} is then negatively related to the expected policy rate and to the expected bills-to-gdp ratio.*

Proof. See Appendix A.2. ∎

Under a binding collateral constraint, the supply of money is bounded and the households' consumption choice is restricted by the available amount of collateral. The associated consumption growth rate as well as the expected inflation rate then determine the willingness to pay for an extra unit of money, i.e. the Euler equation rate. The central bank can in fact implement this equilibrium (see first part of Proposition 1), by setting the policy rate, i.e. the price of money in open market operations, below the Euler equation rate (see 21), which is here guaranteed by $E_t R_{t+1}^m < \pi/\beta$. If, in contrast, it sets the policy rate equal to the Euler equation rate, the price of money equals the households' marginal willingness to pay for cash, implying that the collateral constraint is slack.

As summarized in the second part of Proposition 1, the spread between the Euler equation rate and the policy rate is negatively related to the expected policy rate and the expected bills-to-gdp ratio in the simplified model. A binding collateral constraint, $m_t^R = b_t/R_t^m$ and a binding cash constraint, $y_t = m_t^R$, directly equate the policy rate and the bills-to-gdp rate. Hence, both negative correlations rely on the property that the Euler equation rate moves less than one for one with the policy rate. In the subsequent analysis, we will show that the model generates negative correlations of the liquidity premium with the bills-to-gdp ratio and the policy rate that are also quantitatively consistent with the empirical evidence provided in Section 2.

Monetary policy effects   Under a binding collateral constraint, the model's monetary transmission mechanism differs from the well known logic of New Keynesian models, since the (real) policy rate does not directly govern intertemporal substitution according to a standard consumption Euler equation. For a given beginning-of-period nominal stock of eligible assets, a higher policy rate (i.e. an increase in the price of money in terms of eligible assets) tends to reduce the amount of money that can be acquired via open market operations. Hence, an increase in the policy rate exerts a contractionary effect on nominal expenditures by making money more expensive.

To examine the monetary policy effects, we consider both cases of flexible prices ($\phi = 0$) and sticky prices ($\phi > 0$). For the flexible price case, we extend the analysis on which Proposition 1 is based upon. For the sticky price case, we apply a local approximation to the model at a steady state with a binding collateral constraint, which requires the central bank to set the policy rate target below π/β while restricting changes in the policy rate to be sufficiently small such that $E_t R_{t+1}^m < \pi/\beta$ (see Proposition 1). We then solve the set of equilibrium conditions, which are log-linearized at this steady state.[29] To simplify the analysis, we assume that the central bank sets the policy rate in an exogenous way, i.e. $\rho_y = \rho_\pi = 0$ (see 19) and we consider a sufficiently large degree of inertia, $\rho_R > 1/2$, given that the expected policy rate affects the liquidity premium of treasuries. It should be noted that the equilibrium exhibits determinacy under a large set of feedback coefficients of the interest rate rule, including exogenously set policy rates (see Lemma 1 in Appendix A.2).[30] For the quantitative analysis in Section 5, we consider a more realistic monetary policy and apply an endogenous policy rate rule, which satisfies the Taylor-principle. The following proposition summarizes main effects of changes in the policy rate.

Proposition 2 *Consider a simplified model version with $\phi \geq 0$, $h = 0$, $\alpha = 1$, $\Omega \to \infty$, $\sigma = 1$, and a monetary policy satisfying (19) with $\rho_\pi = \rho_y = 0$, $\rho_R > 1/2$, $E_t R_{t+1}^m < \pi/\beta$, and $\pi = 1$.*

1. *A rise in the policy rate leads on impact to a fall in output and inflation and to a rise in the Euler equation rate.*

2. *The spread s_t^{Euler} decreases with the policy rate, is negatively related to the bills-to-gdp ratio, and is more volatile than the (net) Euler equation rate.*

Proof. See Appendix A.2. ∎

As summarized by Proposition 2, monetary policy exerts conventional macroeconomic effects (e.g. a higher policy rate lowers output and inflation) for flexible and sticky prices. In both cases, the Euler equation rate increases with the policy rate by less than one for one such that the spread s_t^{Euler} decreases in response to a policy rate increase, which accords to our VAR (see Section 2). The simple reason is that the liquidity value of treasuries, which originates in their convertibility into means of payments in open market operations, falls when the (expected) policy rate rises, since the price of money in terms of treasuries in open market operations increases (which requires more treasuries for a particular amount of money). At the same time, the willingness to transfer means of payment to the future is reduced, implying a rise in the Euler equation rate. The liquidity premium on treasuries further falls when more eligible assets are available, i.e. when the bills-to-gdp ratio b_t/y_t increases. Given that money is only supplied via repos, all T-bills are held the

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[29] To derive the unique solution under sticky prices, we apply the local determinacy conditions of the model, which are summarized in Lemma 1 in Appendix A.2.

[30] The reason why local equilibrium determinacy does not depend on the Taylor-principle is that the stock of eligible securities serves as a nominal anchor (like under a money growth policy). This closely relates to the determinacy property of Adao et al.'s (2003) cash-in-advance model with sticky prices, where both, the nominal interest rate and the supply of money, are controlled by the central bank at the same time.

private sector, such that $b_t/y_t = b_t^T/y_t$, which will not be the case in the calibrated version (see Section 5). Moreover, the observation that the Euler spread s_t^{Euler} is more volatile than the (net) Euler equation rate $R_t^{Euler} - 1$ (see Section 2.2 or Chari et al., 2009) is also implied by the model (see part 2 of Proposition 2).[31] Hence, for the simplified version of the model we can summarize that the implied liquidity premium, i.e. spread between the Euler equation rate and the policy rate, behaves in a way that is qualitatively consistent with the evidence provided in Section 2.

5   Quantitative analysis

In this Section, we apply a calibrated version of model and present quantitative results. For this, we solve the model at a steady state where the collateral constraint (2) is binding and consider aggregate shocks that are sufficiently small to remain in a neighborhood of the steady state, such that the collateral constraint is always binding. In the first part of this Section, we describe how the parameter of the model are set. In the second part, the moments of simulated series are compared with the corresponding empirical moments of Section 2, which do not serve as targets for the model calibration. In the third part, we demonstrate that the model is able to generate macroeconomic effects of monetary policy shocks, which are consistent with broad empirical evidence. In the last part of this Section, we assess the robustness of the results by applying an alternative money demand specification.

5.1   Calibration

For most of the parameters we apply standard values, which accord to an interpretation of a period as a quarter. We adopt Christiano et al.'s (2005) values for their non-estimated parameters and set the inverses of the elasticities of intertemporal substitution at $\sigma = 1$ and $\sigma_n = 1$, the labor income share at $\alpha = 2/3$, and the depreciation rate at $\delta = 0.025$. For the fraction of non-optimally price adjusting firms ϕ, and the elasticity of substitution ϵ we chose the values $\phi = 0.8$ and $\epsilon = 6$, and the utility parameter θ is chosen to lead to a steady state working time of $n = 1/3$. While the investment adjustment cost parameter ξ is typically identified by model estimates based on aggregate data (see e.g. Christiano et al., 2005, or Smets and Wouters, 2007), we apply a benchmark value of $\xi = 0.065$ that accords to Groth and Khan's (2010) estimates based on disaggregate data, which is substantially smaller than estimates based on aggregate data (see e.g. $\xi = 2.48$ in Christiano et al., 2005). For a sensitivity analysis, we vary the values of the cost parameter ξ and of the habit parameter h, for which we apply a benchmark value of 0.7 (see Smets and Wouters, 2007).

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[31] For the case of flexible prices it can further be shown that a higher variance of the policy rate reduces the liquidity premium as well (see proof of proposition 2), since the liquidity value of bonds for open market operations becomes more uncertain. Put differently, when the costs associated with the liquidation of bonds get more uncertain, the compensating interest rate increases. This effect accords to the idea of a liquidity *risk* premium (see also Acharya and Pedersen, 2005).

For the policy rate, which is identified with the Federal Funds rate, we set the average value equal to the sample mean of the Federal Funds rate for the sample 1966-2007, $R^m = 1.065^{1/4}$. The inflation target is set equal to the mean inflation rate of the same sample period, $\pi = 1.046^{1/4}$.[32] For the coefficients of the central bank interest rate rule (19), we apply Mehra and Minton's (2007) estimates, which accord to standard values: $\rho_R = 0.73$, $\rho_\pi = 1.5$, $\rho_y = 0.78^{1/4}$, and $sd(\varepsilon_R) = 0.003$. To identify the value for the discount factor β, we use that the model's predictions can be related to observable spreads, like the corporate bond yield spread $R_t^L - R_t$. We decided to set β at an intermediate value, $\beta = 0.993$, which implies that the steady state spread $R^L - R$ equals 0.0025 (where we used $R^L = R^{Euler} = \pi/\beta$ and $R = R^m$), or 100 basis points for annualized rates, which accords to the (AAA) corporate bond yield spread in Krishnamurthy and Vissing-Jorgensen (2012). It further leads – when applying a second-order approximation of the model – to an equity premium $E_0(R_t^q - R_t)$ of 2.32% per annum, implying that our model fails to fully explain the equity premium (which is not the purpose of the paper). We estimate the growth rate Γ of T-bills (see 17) using data for the total stock of T-bills for 1966-2007 from the U.S. Treasury. The estimated value equals $\Gamma = 7.2\%$, which almost exactly equals the growth rate of nominal GDP. Given that we abstract from real growth, we can safely set Γ equal to the inflation target, $\Gamma = \pi$.

The remaining parameters are set in accordance with some empirical observations regarding GDP. We further set the policy parameter Ω equal to 25 to match the maximum output response to monetary policy shocks (i.e. a decrease in output by about 0.25% in deviations from its steady state value in response to an increase in the policy rate by 30 $b.p.$) as identified in the VAR applied in Section 2.2. The autocorrelation coefficient of the AR1-process for total factor productivity (TFP) is set equal to 0.8, while the standard deviation of innovations $\varepsilon_{a,t}$ is calibrated to match the observed standard deviation of hp-filtered GDP for 1966-2007, i.e. $st.dev.((y_t - y)/y) = 1.53$, leading to $sd(\varepsilon_a) = 0.1125$. The parameter values are summarized in Table A1 of Appendix B.

5.2 Selected moments

In this Section, we examine selected moments of interest rates and spreads, which correspond to the moments presented in Table 1 in Section 2. Specifically, we consider the liquidity premium $s_t^{Euler} = R_t^{Euler} - R_t^m$, which has been estimated in Section 2.2, and the spread between corporate and government bonds $s_t^{Treas} = R_t^L - R_t$, whose empirical counterpart has been discussed in Section 2.1. Note that these two spreads are closely related to each other (with a correlation of 0.97), since the borrowing rate of firms R_t^L equals the Euler equation rate, $R_t^L = R_t^{Euler}$, and the treasury rate R_t equals the expected policy rate up to first order (see 12):

$$R_t \approx E_t R_{t+1}^m,$$

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[32]Data for the Federal Funds rate and the inflation rate are taken from FRED database.

Table 2: Unconditional moments of selected series

	Data	W/o Habits h=0			Habits h=0.7	MIU
		MP & TFP	MP	TFP	MP & TFP	MP & TFP
$corr\left(s_t^{Treas}, b_{t-1}/y_t\right)$	-0.62	-0.88	-0.99	-0.85	-0.82	-0.30
$corr\left(R_t, R_t^m\right)$	0.99	0.98	0.99	0.98	0.97	0.99
$corr(s_t^{Euler}, R_t^m)$	-0.82	-0.95	-0.99	-0.90	-0.91	-0.99
$corr\left(R_t^{Euler}, R_t^m\right)$	0.53	0.53	0.69	0.59	0.54	0.79
$sd(s_t^{Euler})/sd(R_t^m)$	0.85	0.88	0.92	0.83	0.86	0.90
$sd(s_t^{Euler})/sd(R_t^{Euler})$	1.49	2.58	7.2	1.83	2.00	9.0

Note: Standard deviations refer to net interest rates and the abbreviations MP, TFP, and MIU denote monetary policy shocks, technology shocks, and a money-in-the-utility-function versions with habits.

which accords to empirical evidence on the short-term treasury rate, see e.g. Simon (1990) and Lange et al. (2003). Table 2 presents correlations between interest rates and the spreads with the bills-to-gdp ratio and with the policy rate, and relative standard deviations. To facilitate comparisons with the empirical results in Section 2, we focus on the model version without habits ($h = 0$); the latter hardly being relevant for the main results of the paper (see Section 5.3). The first column presents the empirical moments (from Table 1), for convenience. The following three columns refer to a model specification without habits ($h = 0$), the next column presents moments of simulated series with external habits ($h = 0.7$), and the last column refers to an alternative model version with money in the utility function (see Section 5.4).

The correlation between the treasury spread s_t^{Treas} and the ratio of T-bills (held by private agents) to gdp b_{t-1}/y_t that is computed from simulated series is also negative though it tends to exceed its empirical counterpart. The correlation for the version with habits comes closest but is still substantially larger than found empirically, which is mainly due to the stylized cash-in-advance specification of money demand. The correlation between the treasury rate and the policy rate almost equals unity, which accords to the data. The correlation between the Euler spread and the policy rate $corr(s_t^{Euler}, R_t^m)$ as well as the correlation between the Euler equation rate and the policy rate $corr\left(R_t^{Euler}, R_t^m\right)$ exhibit the same signs and similar magnitudes compared to their empirical counterparts. Both correlations tends to be larger in absolute terms when only interest rate shocks are considered (which is also implied by Proposition 2). Regarding the relative volatilities, Table 2 shows that the Euler spread s_t^{Euler} exhibits a smaller standard deviation than the policy rate, whereas it is more volatile than the Euler equation rate. The magnitude of former relative standard deviation is close to its empirical counterpart (when both shocks are present), while the latter tends to be larger in the model.

Overall, the model is able to broadly replicate the empirical moments of interest rates and spreads, which standard models entirely fail to explain. Notably, the moments of simulated series

are not substantially altered when habits are taken into account, which is consistent with the impulse response analysis in the subsequent section.

5.3 Monetary transmission

In this Section, we examine the dynamics of interest rates and spreads in response to monetary policy shocks. We further show that responses of macroeconomic aggregates to policy rate shocks accord to conventional expectations about macroeconomic effects,[33] even though the transmission mechanism differs from monetary transmission in standard New Keynesian models, where the real policy rate equals the marginal rate of intertemporal substitution. Figure 1 presents the impulse responses of interest rates and macroeconomic aggregates to a one standard deviation innovation to the policy rate, $\varepsilon_{r,t} > 0$ (see 19). Note that interest rates, the inflation rate, and the bills-to-gdp ratio are presented as absolute deviations from their steady state values, while output, consumption, and investments are presented in percentage deviations from their steady state values, e.g. $\widehat{y}_t = 100 * (y_t - y)/y$. The black solid line shows the impulse responses of the model version with habits, $h = 0.7$, and investment adjustment costs with $\xi = 0.065$. To illustrate that monetary transmission is not mainly driven by intertemporal substitution of consumption and investments, we consider two additional versions: The red solid circled line presents impulse responses without habits, $h = 0$, and the blue dashed line with diamonds presents responses for investment adjustment costs that are twice as large, $\xi = 0.13$.

An increase of the policy rate from its steady state value leads to a smaller rise in the treasury rate R_t, since it is related to the expected future policy rate (see 12). Output and inflation decrease, which imply – together with the supply of T-bills (17) – that the bills-to-gdp ratio b_{t-1}/y_t increases with the policy rate. Notably, these responses are virtually not affected by habit formation or investment adjustment costs. The Euler equation rate rises on impact and returns back to its steady state value from below. The increase in the Euler equation rate relative to the policy rate after a monetary policy shock corresponds to the empirical relative increase, where the maximum increase of the Euler equation rate is about one fifth of the size of the interest rate shock. The initial decrease of the spread $R_t^{Euler} - R_t^m$ after a monetary policy shock represents about two thirds of the shock, which is slightly smaller than the empirical decrease of the spread (88% of the shock), given that empirically the Euler equation rate keeps rising for a few quarters after the shock.

The behavior of the Euler equation rate in the model is reflected by the response of consumption, which grows after it falls on impact. Omitting habits (see red solid circled line), leads to a more pronounced impact effect on consumption and slightly reduced Euler equation rate response. When investment adjustment costs are twice as large (see blue dashed line with diamonds), the maximum investment response is (slightly) reduced, which is compensated by a more pronounced consumption

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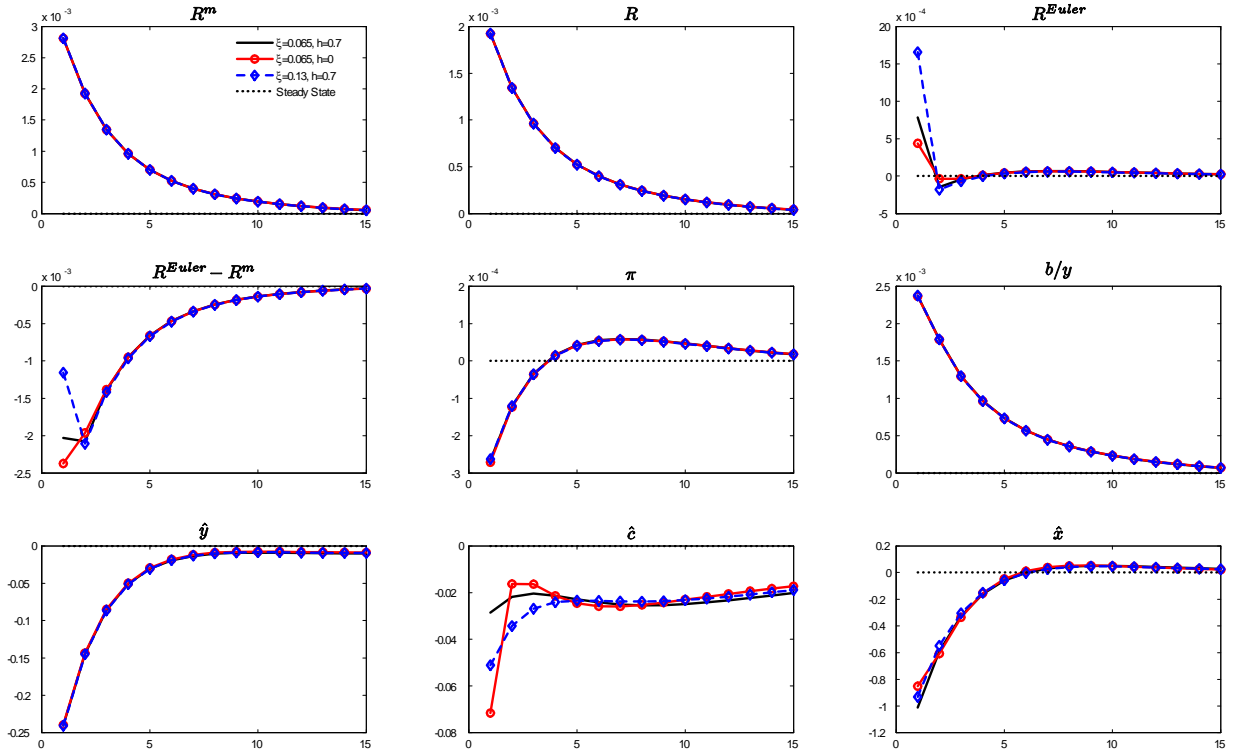[33]Impulse responses a technology shocks can be found in Appendix B.

Figure 1: Impulse responses to policy rate shocks

response. Accordingly, the initial increase in the Euler equation rate is more than twice as large than in the benchmark case. These experiments show that habits and investment adjustment costs mainly alter the composition of aggregate demand, implying that the output and inflation effects of monetary policy are not primarily governed by intertemporal substitution effects. It should be noted that the insensitivity of the output response is mainly due to the money demand specification, which implies that aggregate demand is restricted by $c_t + x_t \geq m_t^H + m_t^R$.

In this model, a higher policy rate predominantly impacts on the level of consumption and investments due to the increased price of money in open market operations and the binding cash constraints. In contrast, in a standard model, where the policy rate equals the Euler equation rate, a change in the policy rate immediately affects the intertemporal consumption and investment choice. Here, part of an increase in the policy rate is reflected by a decrease in liquidity premium (see 8) such that consumption growth is not one-for-one affected by the real policy rate. Likewise, the rate of return on investments is not directly linked to the latter, such that the investment response is less pronounced – for a given magnitude of investment adjustment costs – than in standard models.[34]

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[34]Note that a magnitude of investment adjustment costs that is obtained from estimates based on aggregate data, e.g. $\xi = 5.88$ in Smets and Wouters (2007), is much larger and would here lead to an investment response that is

5.4 Alternative money demand specification

In this Section, we briefly examine an alternative money demand specification to assess the robustness of our main results. Given that the households' and firms' cash constraints (3) and (14) are rather rigid and imply an unrealistic velocity, we apply a widely used money-in-the-utility function (MIU) specification. Specifically, we follow Christiano et al. (2005) and assume that real balances enter household $i's$ utility function in a separable way:

$$u\left(c_{i,t}, M_{i,t}/P_t, n_{i,t}\right) = \frac{(c_{i,t} - hc_{t-1})^{1-\sigma}}{1-\sigma} - \theta\frac{n_{i,t}^{1+\sigma_n}}{1+\sigma_n} + \nu\frac{(M_{i,t}/P_t)^{1-\sigma_m}}{1-\sigma_m}, \qquad (22)$$

where $\sigma_m \geq 1$, $\nu > 0$, and $M_{i,t} = I_{i,t} + M_{i,t-1}^H$. Access to money is still constrained by the collateral constraint (2), whereas the cash constraints (3) and (14) do not apply. Household $i's$ demand for additional money $I_{i,t}$ and holdings of money $M_{i,t}^H$ then satisfy $(R_t^m - 1)\lambda_{i,t} + R_t^m\eta_{i,t} = u_{i,mt}$ and $\beta E_t\left(u_{i,m,t+1}/\pi_{t+1}\right) = \left[\left(R_t^{Euler} - 1\right)/R_t^{Euler}\right]u_{i,ct}$ instead of (7) and (8), where the Euler equation rate measures the opportunity costs of holding money (see discussion in Section 3.4). Like households, firms do not to rely on cash for goods purchases, such that the set of equilibrium conditions changes by $\nu\beta E_t[\left(m_{t+1}^H + m_{t+1}^R\right)^{-\sigma_m}/\pi_{t+1}] = (1 - 1/R_t^{Euler})u_{c,t}$, $\varrho_t = y_t - w_t n_t - x_t$, $\lambda_t = u_{c,t}$, and $1 = q_t[G_t + \frac{x_t}{x_{t-1}}G_t'] - E_t\beta(\frac{\lambda_{t+1}}{\lambda_t}q_{t+1}(\frac{x_{t+1}}{x_t})^2 G_{t+1}')$, which replace the conditions (23) and (32)-(34) given in Definition 1 in Appendix A.1. The parameters ν and σ_m in (22) are calibrated to get a velocity $\overline{y}/\overline{m}$ of 0.44 (see Christiano et al., 2005) and to replicate the impact output effect of policy rate shocks for the benchmark parametrization ($\nu = 270$ and $\sigma_m = 10$). The standard deviation of TFP shocks is again adjusted to match the observed standard deviation of detrended output ($st.dev.(\varepsilon_a) = 0.0065$), while the remaining parameter values are unchanged.

The model is again solved for the case of a binding collateral constraint.[35] When money demand is induced by a MIU specification, the qualitative results with regard to interest rates and spreads are unchanged. Like before, we compute selected moments of simulated time series taking both shocks, i.e. policy rate shocks and technology shocks, into account. The last column in Table 2 shows that all correlations exhibit the same sign and similar magnitudes as their empirical counterparts, though the negative correlation between the spread s_t^{Treas} and the bills-to-gdp ratio is less pronounced. The standard deviation of the Euler spread again lies between the standard deviation of the Euler equation rate and the policy rate, while the latter ratio is much larger than in the benchmark model. The impulse responses to an innovation to the policy rate are presented in Figure 2.[36] They are consistent with results presented above and broadly show the same pattern as the benchmark model (see Figure 1). Given that goods purchases are now not tightly restricted by money supply, aggregate demand and inflation are now affected by habit

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smaller than the output response, which is clearly at odds with VAR evidence (see, e.g. Christiano et al., 2005).

[35]In the steady state, the multiplier on collateral constraint now satisfies $\eta/u_c = R^{Euler}[(1/R^m) - (1/R^{Euler})]$, such that $R^{Euler} > R^m$ again implies a binding collateral constraint.

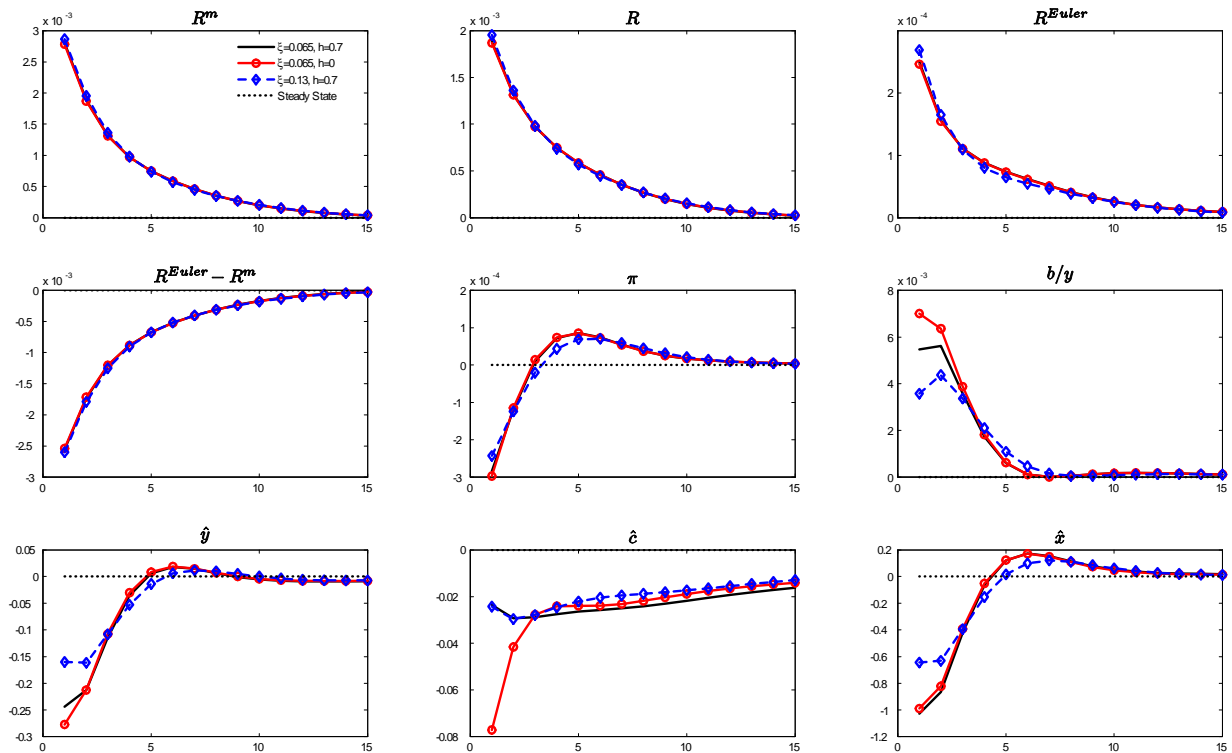[36]Responses to productivity shocks are again provided in Appendix B.

Figure 2: Impulse responses to policy rate shocks for an alternative money demand

formation and investment adjustment costs, though to a small extent, while response of the Euler equation rate is now virtually unchanged.

6 Conclusion

In this paper, we present a simple macroeconomic model where the rate of return on short-term treasuries is endogenously linked to the monetary policy rate and tends to be smaller than the rates on corporate borrowing, consistent with broad empirical evidence. We introduce monetary policy implementation via open market operations into a standard macroeconomic model, which gives rise to a liquidity premium on eligible assets, i.e. short-term treasuries, compared to non-eligible assets. The model predicts that this liquidity premium is negatively related to the ratio of bills to GDP, which accords to empirical evidence. While standard macroeconomic models typically assume that the (real) policy rate equals the marginal rate of intertemporal substitution, we show that they differ and that the spread – also known as the Euler equation error – is negatively related to the policy rate and more volatile that the consumption Euler equation rate, which has been reported in several studies. Although the existence of a liquidity premium substantially alters the monetary transmission mechanism, compared to a standard New Keynesian model for example, responses of real activity and inflation to monetary policy shocks are consistent with broad empirical evidence.

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A Appendix

A.1 Equilibrium conditions

Definition 1 *A rational expectations equilibrium (REE) is a set of sequences $\{c_t, y_t, n_t, x_t, k_t,$ $w_t, \varrho_t, v_t, q_t, \lambda_t, m_t^R, m_t^H, b_t, b_t^T, mc_t, Z_{1,t}, Z_{2,t}, Z_t, s_t, \pi_t, R_t, R_t^{Euler} = R_t^L, R_t^q\}_{t=0}^\infty$ satisfying*

$$c_t + x_t = m_t^H + m_t^R, \ if \ R_t^{Euler} > 1, \quad or \ c_t + x_t \le m_t^H + m_t^R, \ if \ R_t^{Euler} = 1, \quad (23)$$

$$b_{t-1}/\left(R_t^m \pi_t\right) = m_t^H - m_{t-1}^H \pi_t^{-1} + m_t^R, \ if \ R_t^{Euler} > R_t^m, \tag{24}$$

$$or \ b_{t-1}/\left(R_t^m \pi_t\right) \ge m_t^H - m_{t-1}^H \pi_t^{-1} + m_t^R, \ if \ R_t^{Euler} = R_t^m,$$

$$E_t u_{c,t+1} \pi_{t+1}^{-1} = R_t E_t \left(R_{t+1}^m\right)^{-1} u_{c,t} \pi_{t+1}^{-1}, \tag{25}$$

$$m_t^R = \Omega m_t^H, \tag{26}$$

$$\theta n_t^{\sigma_n} = u_{c,t} w_t / R_t^{Euler}, \tag{27}$$

$$1/R_t^{Euler} = \beta E_t \left[u_{c,t+1}/\left(u_{c,t} \pi_{t+1}\right)\right], \tag{28}$$

$$w_t = mc_t \alpha a_t n_t^{\alpha-1} k_{t-1}^{1-\alpha}, \tag{29}$$

$$R_t^q = P_t \left(v_t + \varrho_t\right)/\left(P_{t-1} v_{t-1}\right), \tag{30}$$

$$1 = \beta E_t \left[(\lambda_{t+1}/\lambda_t) \cdot \left(R_{t+1}^q/\pi_{t+1}\right)\right]. \tag{31}$$

$$\varrho_t = y_t - w_t n_t - x_t (2R_t^{Euler} - 1)/R_t^{Euler}, \tag{32}$$

$$\lambda_t = \beta E_t \left[(u_{c,t+1}/\pi_{t+1}\right], \tag{33}$$

$$R_t^{Euler} = q_t \left[G_t + (x_t/x_{t-1}) G_t'\right] - E_t \beta \left[(\lambda_{t+1}/\lambda_t) q_{t+1} (x_{t+1}/x_t)^2 G_{t+1}'\right], \tag{34}$$

$$q_t = \beta E_t \left[(\lambda_{t+1}/\lambda_t) \left((1-\alpha)mc_{t+1} \left(y_{t+1}/k_t\right) + (1-\delta) q_{t+1}\right)\right], \tag{35}$$

$$Z_{1,t} = \lambda_t y_t mc_t + \phi \beta E_t \pi_{t+1}^\varepsilon Z_{1,t+1}, \tag{36}$$

$$Z_{2,t} = \lambda_t y_t + \phi \beta E_t \pi_{t+1}^{\varepsilon-1} Z_{2,t+1}, \tag{37}$$

$$Z_t = \left[\varepsilon/\left(\varepsilon - 1\right)\right] Z_{1,t}/Z_{2,t}, \tag{38}$$

$$1 = (1 - \phi)Z_t^{1-\varepsilon} + \phi \pi_t^{\varepsilon-1}, \tag{39}$$

$$s_t = (1 - \phi)Z_t^{-\varepsilon} + \phi s_{t-1} \pi_t^\varepsilon, \tag{40}$$

$$y_t = a_t n_t^\alpha k_{t-1}^{1-\alpha}/s_t, \tag{41}$$

$$y_t = c_t + x_t, \tag{42}$$

$$k_t = (1 - \delta) k_{t-1} + x_t G_t, \tag{43}$$

$$b_t = b_{t-1} \pi_t^{-1} + b_t^T - b_{t-1}^T \pi_t^{-1} - \left(m_t^H - m_{t-1}^H \pi_t^{-1}\right), \tag{44}$$

$$b_t^T = \Gamma b_{t-1}^T/\pi_t, \tag{45}$$

(where $u_{c,t} = (c_t - hc_{t-1})^{-\sigma}$, $G_t = 1 - \xi\frac{1}{2}(x_t/x_{t-1} - 1)^2$, $G_t' = -\xi(x_t/x_{t-1} - 1)$) and the transversality conditions, a monetary policy setting $\{R_t^m \ge 1\}_{t=0}^\infty$ according to (19), $\Omega_t > 0$, and $\pi \ge \beta$, and a fiscal policy setting $\Gamma \ge 1$, for given sequences $\{a_t, \varepsilon_{r,t}\}_{t=0}^\infty$ and $\{\widetilde{y}_t\}_{t=0}^\infty$ (see below), and initial values $M_{-1}^H > 0$, $B_{-1} > 0$, $B_{-1}^T > 0$, $k_{-1} > 0$, $x_{-1} > 0$, and $s_{-1} \ge 1$.

The efficient output level $\widetilde{y}_t$, which required for the interest rate rule (19), is jointly determined with the efficient allocation $\{\widetilde{y}_t, \widetilde{n}_t, \widetilde{c}_t, \widetilde{k}_t, \widetilde{x}_t, \widetilde{q}_t\}_{t=0}^\infty$ satisfying

$$\theta\widetilde{n}_t^{1+\sigma_n} = \widetilde{u}_{ct}\alpha\widetilde{y}_t, \quad \widetilde{y}_t = a_t\widetilde{n}_t^\alpha\widetilde{k}_{t-1}^{1-\alpha}, \quad \widetilde{y}_t = \widetilde{c}_t + \widetilde{x}_t, \quad \widetilde{k}_t = (1-\delta)\widetilde{k}_{t-1} + \widetilde{x}_t G\left(\widetilde{x}_t/\widetilde{x}_{t-1}\right),$$

as well as

$$1 = \widetilde{q}_t \left[G\left(\widetilde{x}_t/\widetilde{x}_{t-1} \right) + \left(\widetilde{x}_t/\widetilde{x}_{t-1} \right) G'\left(\widetilde{x}_t/\widetilde{x}_{t-1} \right) \right] - E_t \beta \left[\left(\widetilde{u}_{c,t+1}/\widetilde{u}_{c,t} \right) \widetilde{q}_{t+1} \left(\widetilde{x}_{t+1}/\widetilde{x}_t \right)^2 G'\left(\widetilde{x}_{t+1}/\widetilde{x}_t \right) \right],$$

$$\widetilde{q}_t = \beta E_t \left[\left(\widetilde{u}_{c,t+1}/\widetilde{u}_{c,t} \right) \left((1-\alpha)(\widetilde{y}_{t+1}/\widetilde{k}_t) + (1-\delta) \widetilde{q}_{t+1} \right) \right],$$

where $\widetilde{u}_{ct} = (\widetilde{c}_t - h \cdot \widetilde{c}_{t-1})^{-\sigma}$ given $\widetilde{x}_{-1} > 0$ and $\widetilde{k}_{-1} > 0$. If the collateral constraint is not binding, which would be the case when the policy rate equals the Euler equation rate, $R_t^m = R_t^{Euler}$ (see 3.4), the model as given in Definition 1 can be reduced to a conventional sticky price model with a cash-credit good distortion, where Ricardian equivalence holds and money holdings can separately be determined by (23) and (26) if $R_t^{Euler} > 1$.

Definition 2 *When the collateral constraint (20) is not-binding, a REE is a set of equilibrium sequences $\{c_t, y_t, n_t, x_t, k_t, w_t, \varrho_t, v_t, q_t, \lambda_t, mc_t, Z_{1,t}, Z_{2,t}, Z_t, s_t, \pi_t, R_t, R_t^{Euler} = R_t^L, R_t^q\}_{t=0}^{\infty}$ satisfying (27)-(43), $1/R_t^m = \beta E_t \left[u_{c,t+1}/ \left(u_{c,t} \pi_{t+1} \right) \right]$, and $R_t = R_t^m$, for a monetary policy setting $\{R_t^m \geq 1\}_{t=0}^{\infty}$ according to (19), $\pi \geq \beta$, for given sequences $\{a_t, \varepsilon_{r,t}\}_{t=0}^{\infty}$ and $\{\widetilde{y}_t\}_{t=0}^{\infty}$, and initial values $k_{-1} > 0$, $x_{-1} > 0$, and $s_{-1} \geq 1$.*

A.2   Appendix to Section 4

In this Appendix, we first define a *REE* under flexible prices for the simplified version, before we prove the claims made in Proposition 1. Then, we examine the local determinacy properties under sticky prices, which will be used for the subsequent proof of Proposition 2. For the specific parameter values $\phi = 0$, $h = 0$, $\alpha = 1$, $\Omega \to \infty$, $\sigma = 1$, and $\Gamma = \pi = 1$, the model as given in Definition 1 can be simplified as follows.

Definition 3 *For $\phi = h = 0$, $\alpha = \sigma = \Gamma = \pi = 1$, and $\Omega \to \infty$, a REE is a set of sequences $\{c_t, n_t, y_t, w_t, m_t^R, \pi_t, R_t^{Euler}, b_t\}_{t=0}^{\infty}$ and $P_0 > 0$ satisfying (27)-(28),*

$$c_t = m_t^R, \text{ if } R_t^{Euler} > 1, \quad \text{or } c_t \leq m_t^R, \text{ if } R_t^{Euler} = 1, \tag{46}$$

$$b_{t-1}/ \left(R_t^m \pi_t \right) = m_t^R, \text{ if } R_t^{Euler} > R_t^m, \quad \text{or } b_{t-1}/ \left(R_t^m \pi_t \right) \geq m_t^R, \text{ if } R_t^{Euler} = R_t^m, \tag{47}$$

$$b_t = \pi b_{t-1} \pi_t^{-1}, \tag{48}$$

$$y_t = a_t n_t, \tag{49}$$

$w_t = a_t(\varepsilon - 1)/\varepsilon$, $y_t = c_t$, and the transversality conditions, for a monetary policy setting $\{R_t^m \geq 1\}_{t=0}^{\infty}$ according to (19) with $\widetilde{y}_t = a_t^{\frac{1+\sigma_n}{\sigma+\sigma_n}} \theta^{\frac{-1}{\sigma+\sigma_n}}$, $\Omega_t > 0$, and $\pi \geq \beta$, given $\{a_t, \varepsilon_{r,t}\}_{t=0}^{\infty}$ and $b_{-1} > 0$.

Proof of proposition 1.   Consider the model summarized in Definition 3. Combining (27), $y_t = c_t$, and (49) leads to $y_t = a_t \left[(mc/\theta) \left(1/R_t^{Euler} \right) \right]^{1/(1+\sigma_n)}$, such that a REE can be reduced to

a set of sequences $\{y_t, \pi_t, R_t^{Euler}, b_t\}_{t=0}^{\infty}$ and $P_0 > 0$ satisfying (48),

$$y_t = a_t \left[(\mu/\theta) \left(1/R_t^{Euler} \right) \right]^{1/(1+\sigma_n)}, \tag{50}$$

$$1/R_t^{Euler} = \beta y_t E_t [1/(y_{t+1}\pi_{t+1})], \tag{51}$$

$$y_t = b_{t-1}/\left(R_t^m \pi_t \right), \text{ if } R_t^{Euler} > R_t^m, \text{ or } y_t \leq b_{t-1}/\left(R_t^m \pi_t \right), \text{ if } R_t^{Euler} = R_t^m, \tag{52}$$

and $\pi P_0 b_0 = B_{-1}$, where $\mu = \frac{\varepsilon - 1}{\varepsilon} < 1$, for a monetary policy setting R_t^m and for a given initial stock of treasuries $B_{-1} > 0$. Consider the case where the constraint (52) is binding, which requires $R_t^{Euler} > R_t^m$ to hold in equilibrium according to (21). Eliminating output in (51) with (52) for $R_t^{Euler} > R_t^m$, gives $1/R_t^{Euler} = \beta b_{t-1}(b_t R_t^m \pi_t)^{-1} E_t R_{t+1}^m$, and substituting out b_t by (48) gives

$$1/R_t^{Euler} = (\beta/\pi) E_t R_{t+1}^m / R_t^m \tag{53}$$

$\Leftrightarrow R_t^{Euler}/R_t^m = (\pi/\beta) \left(1/E_t R_{t+1}^m \right)$. The latter implies that if $E_t R_{t+1}^m < \pi/\beta$, the Euler equation rate exceeds the policy rate, $R_t^{Euler} > R_t^m$, which is consistent with a binding collateral constraint, and that the spread R_t^{Euler}/R_t^m tends to decrease with the expected policy rate. It further immediately follows from (48) and (52) that the policy rate is positively related to the bills-to-gdp ratio, $R_t^m = (b_t/y_t)\pi^{-1}$, such that the spread R_t^{Euler}/R_t^m is negatively related to the expected bills-to-gdp ratio $R_t^{Euler}/R_t^m = (\pi/\beta) \left[\pi/E_t \left(b_{t+1}/y_{t+1} \right) \right]$. ∎

Under sticky prices, $\phi > 0$, and for $h = 0$, $\Omega \to \infty$, and $\alpha = \Gamma = \pi = 1$, the model as given in Definition 1 can be reduced to a set of sequences $\{c_t, n_t, y_t, w_t, m_t^R, \pi_t, R_t^{Euler}, b_t, mc_t, Z_{1,t}, Z_{2,t}, Z_t, s_t\}_{t=0}^{\infty}$ and $P_0 > 0$ satisfying (27)-(28), (36)-(40), (46)-(48), $w_t = mc_t a_t$, $y_t = a_t n_t / s_t$, $y_t = c_t$, and the transversality conditions, for a monetary policy setting $\{R_t^m \geq 1\}_{t=0}^{\infty}$ according to (19), $\Omega_t > 0$, and $\pi > \beta$, given a sequence $\{a_t\}_{t=0}^{\infty}$ and initial values $b_{-1} > 0$ and $s_{-1} \geq 1$.

Suppose that the average policy rate and the inflation target satisfy $R^m < \pi/\beta$ and $\pi > \beta \Rightarrow R^{Euler} > 1$, where steady state values exhibit no time index. Then, the collateral constraint is binding in the steady state. Log-linearizing the model at this steady state and assuming that shocks are sufficiently small such that the economy remains in the neighborhood of the steady state, we can define a REE as follows (where $\hat{x}_t$ denotes log-deviations from the steady state value x, $\hat{x}_t = \log x_t/x$).

Definition 4 For $\Omega \to \infty$, $h = 0$, $\alpha = \Gamma = \pi = 1$, $R^m \in [1, 1/\beta)$, a REE is a set of convergent sequences $\{\hat{y}_t, \pi_t, \hat{b}_t, \hat{R}_t^{Euler}, \hat{R}_t^m\}_{t=0}^{\infty}$ satisfying

$$\hat{y}_t = \hat{b}_{t-1} - \hat{\pi}_t - \hat{R}_t^m, \tag{54}$$

$$\sigma \hat{y}_t = \sigma E_t \hat{y}_{t+1} - \hat{R}_t^{Euler} + E_t \hat{\pi}_{t+1}, \tag{55}$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \chi \varpi \hat{y}_t - \chi (1 + \sigma_n) \hat{a}_t + \chi \hat{R}_t^{Euler}, \tag{56}$$

$$\hat{b}_t = \hat{b}_{t-1} - \hat{\pi}_t, \tag{57}$$

$$\hat{R}_t^m = \rho_R \hat{R}_{t-1}^m + \rho_\pi (1 - \rho_R) \hat{\pi}_t + \rho_y (1 - \rho_R) (\hat{y}_t - \varsigma \hat{a}_t) + \varepsilon_{r,t}, \tag{58}$$

where $\varpi = \sigma_n + \sigma > 0$, $\chi = (1 - \phi)(1 - \beta\phi)/\phi$, and $\varsigma = (1 + \sigma_n) / (\sigma_n + \sigma)$, given $b_{-1} > 0$.

The following lemma describes local determinacy for the REE given in definition 4.

Lemma 1 *Suppose that the central bank sets the policy rate according to (19) with $\rho_R = 0$. The REE as given in Definition 4 is uniquely determined if*

$$\left(\rho_\pi + 1/2\right)\left(\sigma_n - \sigma\right) > -\left(1 + \rho_y\right)\left(1 + \beta + \chi\right)\chi^{-1}, \tag{59}$$

Proof. Consider the model given in Definition 4, which can – by eliminating the Euler equation rate with (55) – be further reduced to (54), (57), (58), and

$$\widehat{\pi}_t = \left(\beta + \chi\right)E_t\widehat{\pi}_{t+1} + \chi\left(\varpi - \sigma\right)\widehat{y}_t + \chi\sigma E_t\widehat{y}_{t+1}. \tag{60}$$

Abstracting from shocks, $a_t = 1$ and $\varepsilon_{r,t} = 0$, for simplicity, and using that (58) implies $\widehat{R}_t^m = \rho_\pi\widehat{\pi}_t + \rho_y\widehat{y}_t$, condition (54) leads to $\widehat{y}_t = \frac{1}{1+\rho_y}\widehat{b}_{-1} - \frac{1+\rho_\pi}{1+\rho_y}\widehat{\pi}_t$. Substituting out output with the latter, (60) can together with (57) be written as $[(\beta + \chi) - \chi\sigma\frac{1+\rho_\pi}{1+\rho_y}]E_t\widehat{\pi}_{t+1} + \frac{\chi\varpi}{1+\rho_y}\widehat{b}_t = \widehat{\pi}_t[1 + \chi\left(\varpi - \sigma\right)\frac{\rho_\pi}{1+\rho_y}]$. Hence, the model can be reduced to a two-dimensional system in $\widehat{b}_t$ and $\widehat{\pi}_t$, which exhibits the characteristic polynomial $F(X) = X^2 - \left(\frac{\zeta_2}{\zeta_1} + \frac{\zeta_3}{\zeta_1} + 1\right)X + \frac{\zeta_3}{\zeta_1}$, where $\zeta_1 = (\beta + \chi) - \chi\sigma\frac{1+\rho_\pi}{1+\rho_y}$, $\zeta_2 = \frac{\chi\varpi}{1+\rho_y} > 0$, and $\zeta_3 = 1 + \chi\left(\varpi - \sigma\right)\frac{\rho_\pi}{1+\rho_y} > 0$. Hence, $F(0) = \frac{\zeta_3}{\zeta_1}$ and $F(1) = -\frac{\zeta_2}{\zeta_1}$, implying $signF(0) = -signF(1)$, and there exists at least one real stable eigenvalue between zero and one. Further, $F(X)$ at $X = -1$ is given by $F(-1)\zeta_1/2 = (\beta + 1 + \chi) + \chi\frac{(1/2)+\rho_\pi}{1+\rho_y}(\sigma_n - \sigma)$. For $F(-1)\zeta_1/2 > 0$, such that $signF(0) = signF(-1)$, there exists exactly one stable eigenvalue, between zero and one, and one unstable eigenvalue, indicating local determinacy. Hence, the equilibrium is uniquely determined if (59) is satisfied. ∎

Note that the determinacy condition (59) is hardly restrictive for a reasonable choice of parameter values. The following proof examines the simplified version for flexible prices and for sticky prices, where (59) is satisfied and determinacy is guaranteed by $\sigma = 1$ and $\rho_\pi = \rho_y = 0$.

Proof of proposition 2. Consider a simplified version of the model with $\phi \geq 0$, $h = 0$, $\alpha = 1$, $\Omega \to \infty$, $\sigma = 1$, and a monetary policy satisfying (19) with $\rho_\pi = \rho_y = 0$, $\rho_R > 1/2$, $E_t R_{t+1}^m < \pi/\beta$, and $\pi = 1$.

To establish the claims made in the <u>first</u> part of the proposition, we separately examine the flexible price case and the sticky price case. Consider the model summarized in definition 3. The equilibrium sequences $\{y_t, \pi_t, R_t^{Euler}, b_t\}_{t=0}^\infty$ are characterized by (48), (50)-(51), $\pi P_0 b_0 = B_{-1}$, and $y_t = b_{t-1}/\left(R_t^m\pi_t\right)$. Using that (53) then holds (see proof of Proposition 1) and (19), leads to

$$R_t^{Euler} = (R_t^m)^{1-\rho_R} \cdot (1/R^m)^{1-\rho_R}\left(\beta\exp[(1/2)var(\varepsilon_{r,t})]\right)^{-1}, \tag{61}$$

where we used that $E_t\exp(\varepsilon_{r,t+1}) = \exp[(1/2)var(\varepsilon_{r,t})]$. Next, substitute out the Euler equation rate in (50) with (61), to get

$$y_t = (1/R_t^m)^{\frac{1-\rho_R}{1+\sigma_n}}a_t\Theta, \tag{62}$$

where $\Theta \equiv ((\mu/\theta)(\beta)R^m \exp[(1/2)var(\varepsilon_{r,t})])^{1/(1+\sigma_n)}$. Further, substitute out output with (62) in $\pi_t = b_{t-1}/(R_t^m y_t)$ (see 52), which leads to

$$\pi_t = (1/R_t^m)^{\frac{\sigma_n+\rho_R}{1+\sigma_n}} b_{t-1}/(a_t \Theta). \tag{63}$$

The solutions (61)-(63) imply that output and inflation decrease with the policy rate and that the Euler equation rate increases with the policy rate.

Now consider the sticky price case, summarized in Definition 4 with $\sigma = 1$. Given that the policy rate is exogenous, condition (59) reduces to $\chi(1/2)(1+\sigma_n) > -(1+\beta)$, which is obviously satisfied, implying that the equilibrium is locally determined and the stable eigenvalue is strictly positive (see proof of Lemma 1). Hence, the unique solution to the system (54)-(57), is given by the generic form $\widehat{\pi}_t = \delta_1 \widehat{b}_{t-1} + \delta_2 \widehat{R}_t^m + \delta_5 \widehat{a}_t$, $\widehat{y}_t = \delta_3 \widehat{b}_{t-1} + \delta_4 \widehat{R}_t^m + \delta_6 \widehat{a}_t$, and $\widehat{b}_t = (1-\delta_1)\widehat{b}_{t-1} - \delta_2 \widehat{R}_t^m - \delta_5 \widehat{a}_t$, where the stable eigenvalue is $1-\delta_1 \in (0,1)$ (see Lemma 1). Inserting these solutions into the two-dimensional system (57) and (60) for $\rho_{\pi,y} = 0$ leads to the following conditions for the coefficients δ_2 and δ_4 :

$$\partial\widehat{\pi}_t/\partial\widehat{R}_t^m = \delta_2 = -\chi\left[(1+\sigma_n) - (1-\rho_R)\right]/\Psi < 0,$$
$$\partial\widehat{y}_t/\partial\widehat{R}_t^m = \delta_4 = -\left[1 + \delta_1\beta + \chi(1-\rho_R)\right]/\Psi < 0,$$

where $\Psi = 1 + \delta_1\beta + \chi(1+\sigma_n) > 0$. Hence, in response to a monetary contraction, $\widehat{R}_t^m > 0$ inflation and output decline, while the Euler equation rate, which satisfies (61) and thus $\widehat{R}_t^{Euler} = (1-\rho_R)\widehat{R}_t^m$, increases.

Turning to the <u>second</u> part of the proposition, we use that the solution to the Euler equation rate (61) holds regardless of the degree of price flexibility. It implies for the spread R_t^{Euler}/R_t^m, $R_t^{Euler}/R_t^m = (R_t^m)^{-\rho_R} \cdot (1/R^m)^{1-\rho_R} (\beta \exp[(1/2)var(\varepsilon_{r,t})])^{-1}$. Hence, the ratio R_t^{Euler}/R_t^m decreases with the policy rate and with its variance, while $R_t^m = \pi b_t/y_t$ (or in log-linearized terms $\widehat{R}_t^m = \widehat{b}_t - \widehat{y}_t$) implies that R_t^{Euler}/R_t^m is negatively related to the bills-to-gdp ratio. The variance of the ratio R_t^{Euler}/R_t^m is larger than the variance of the Euler equation rate R_t^{Euler} for a sufficiently large autocorrelation of the policy rate, $\rho_R > 1/2$. Using the approximations $\log(R_t^{Euler}/R_t^m) \approx s_t^{Euler}$ and $\log R_t^{Euler} \approx R_t^{Euler} - 1$, establishes the claims made in the part 2 of the proposition. ∎

B Additional Appendix

Table A1: Benchmark parameter values	
Subjective discount factor	$\beta = 0.993$
Inverse of intertemporal substitution elasticity	$\sigma = 1$
Inverse of Frisch elasticity of labour supply	$\sigma_n = 1$
Substitution elasticity	$\varepsilon = 6$
Steady state working time	$n = 0.33$
Labour share	$\alpha = 0.66$
Investment adjustment cost	$\vartheta = 0.065$
Rate of depreciation of capital stock	$\delta = 0.025$
Habit parameter	$h = 0.7$
Fraction of non-price adjusting firms	$\phi = 0.8$
Steady state interest rate	$R^m = 1.015\,9$
Share of repos to outright purchases	$\Omega = 25$
Steady state inflation	$\pi = 1.011\,3\ (= \Gamma)$
Policy rule coefficients	$\rho_R = 0.73,\ \rho_\pi = 1.5,\ \rho_y = 0.78^{1/4}$
Standard deviation of policy rate shocks	$sd(\varepsilon_{R,t}) = 0.003$
Autocorrelation of TFP-shocks	$\rho_A = 0.8$
Standard deviation of TFP-shocks	$sd(\varepsilon_{A,t}) = 0.01125$

Figure A1: Computed Euler Equation Rate

- —— Euler rate
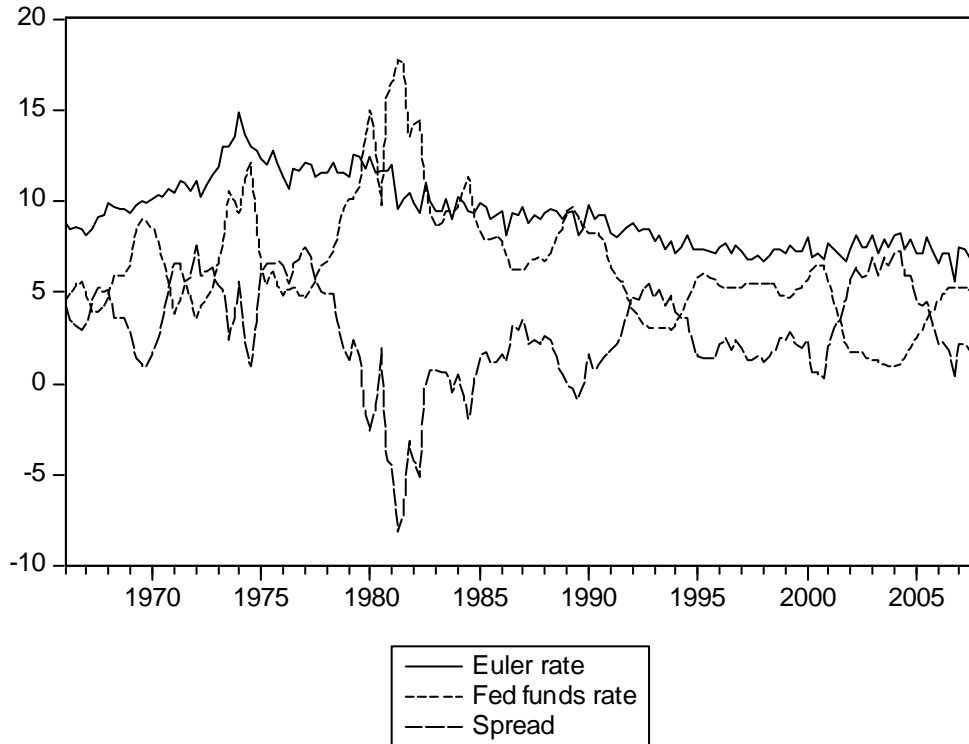- ----- Fed funds rate
- ——- Spread

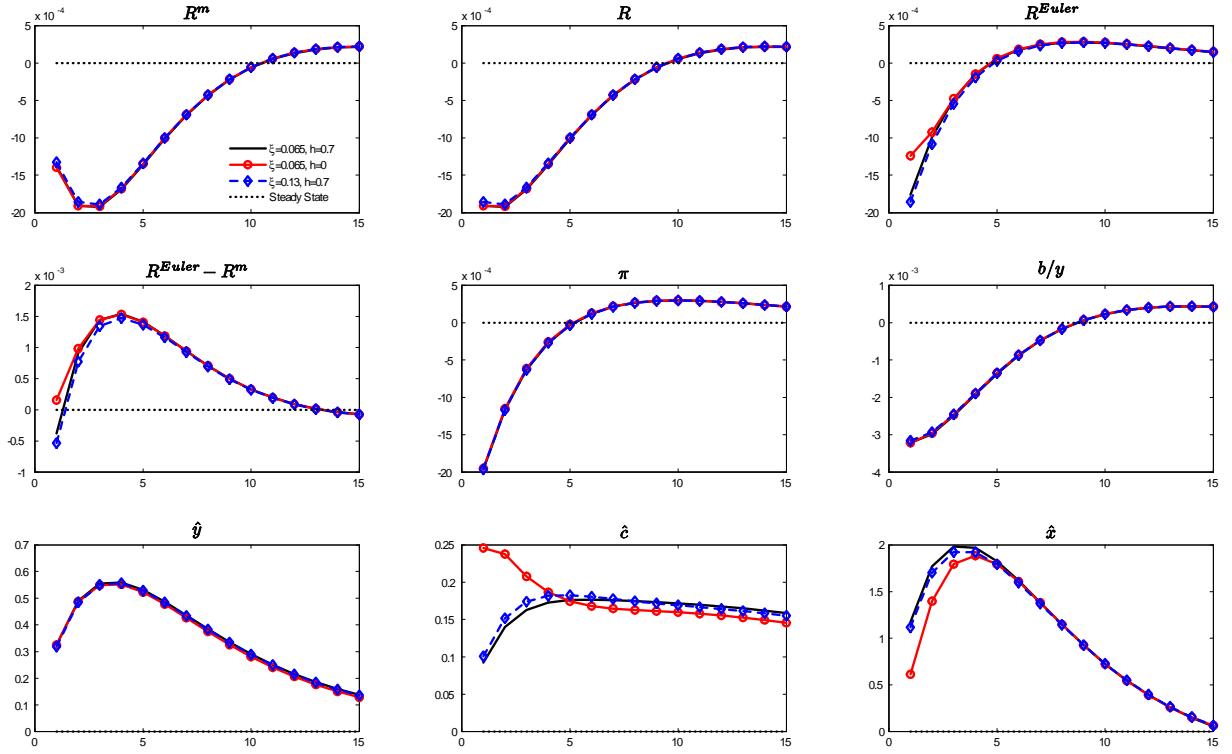Figure A2: Impulse responses to a positive TFP shock

Figure A3: Impulse responses to a positive TFP shock for the MIU version