Optimal government spending with labor market frictions

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This version: July 5, 2011

JEL classification: *E62, E32*

Keywords: *Optimal fiscal policy; government spending; labor market frictions; unemployment.*

Abstract: We study optimal government spending in a business cycle model with labor income taxes and unemployment due to hiring costs. Labor market frictions raise the optimal steady state ratio of government spending to private consumption. The labor tax rate is higher since profits are taxed that arise from employed workers which save hirings costs. For calibrated examples, the quantitative effect of labor market frictions on optimal fiscal policy is small. In the short run, optimal policy involves a strongly procyclical reaction of the tax rate to technology and preference shocks, while the ratio of public to private spending is close to flat. This ratio is, however, markedly countercyclical if taxes are constrained to be constant over the cycle.

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1 Introduction

This paper studies optimal government spending and tax policies in an economy with frictional unemployment. We assume that the government produces public goods that yield utility to private households, and finances its spending by means of a proportional tax on labor income. Unemployment exists because hiring is costly (as in Blanchard and Gali, 2010), such that firms willing to expand their employment have to expend resources that depend on the aggregate tightness of the labor market. Wages are determined through Nash bargaining. Due to the existence **of** hiring costs, there are generally non-zero profits even though firms are competitive and production takes place under constant returns to scale. The government solves a Ramsey problem by choosing sequences of labor tax rates and spending levels that lead to the equilibrium allocation yielding the highest level of welfare.

We analyze in how far the existence of labor frictions matters for optimal fiscal policies with respect to both government spending and labor taxation. We are particularly interested in the optimal relation between public and private consumption. Previous papers have analyzed optimal taxation problems in economies with unemployment for exogenously given government expenditures (see Domeij, 2005, or Arseneau and Chugh, 2009). Further, the joint determination of government spending and taxes has been analyzed by Lansing (1997), but in the context of a standard real business cycle model where labor markets are frictionless. Our paper merges these strands of the literature in that we consider public spending as an instrument of the government that can be used along with distortionary taxation in a setting with frictional unemployment.

Our central results can best be understood in relation to a benchmark case where there are no labor market frictions in the sense that there are no hiring costs and the real wage is competitively determined as the marginal rate of substitution between leisure and consumption (i.e. the benchmark case is a real business cycle model with distortionary labor income taxation). In this case, the optimal fiscal policy would be to choose government spending to equalize the marginal utilities of public and private consumption. With logarithmic utility, the optimal steady state ratio of goverment spending to private consumption is constant, and labor taxation finances this level of expenditures and initial debt. When labor market frictions do exist, however, the steady state ratio of public to private consumption is higher than under optimal policy, and increases with the size of hiring costs and thus of unemployment. At the same time, labor taxes are higher than in the benchmark case. This is first shown analytically for a simplified version of the model where we abstract from capital accumulation, and then confirmed numerically for a calibrated version with physical capital.

We show that this result emerges because hiring costs lead to the existence of profits in equilibrium, whereas government spending and labor taxes in the steady state would be set at the benchmark levels if there were no profits. Profits exist under positive hiring costs, however, since at any point in time the non-separated part of the previous period's employment level is given, which reduces the amount of costly hires that firms need to undertake. To the extent that labor relations last longer than a period, firms thus enjoy economic rents. The government has an incentive to tax these rents. Given that there is no profit tax, labor taxes therefore are higher with labor frictions, and the government partly compensates households for the implied loss in private consumption possibilities by choosing a relatively higher value of utility providing government spending.

Numerically, we find that for plausibly calibrated model versions intended to capture average labor market features of continental European countries in a stylized way, the departure of optimal fiscal policies from the benchmark case in the steady state appears to be quantitatively small. The reason is that the size of labor market frictions depends on the steady state share of the resource costs of hiring in total output, which – following Blanchard and Gali (2010) – are assumed to be about one percent of yearly output. We also present sensitivity analyses to assess the robustness of the results. As it turns out, the departure of the optimal public to private spending ratio and the tax rate from the benchmark case is robustly small (this is also true for different values of workers' bargaining power in wage determination). Thus, the welfare loss from applying policies that would be optimal in the benchmark case to an economy with quantitatively moderate hiring cost frictions appears to be close to negligible.

We further examine the responses under optimal fiscal policy to macroeconomic shocks. A positive technology shock leads to procyclical reactions of employment, government spending, private consumption, and investment, as well as of the ratio of government spending to consumption. However, these responses are quantitatively much smaller in an economy with hiring costs than in the benchmark model, since optimal policy takes the costliness of labor reallocations into account. At the same time, the tax rate response is more pronounced under hiring costs due to the strongly procyclical behavior of profits.

In addition, we consider the case where the tax rate cannot be changed in the short run but is held constant (which we view as a sensible restriction when discussing optimal policy at business cycle frequencies). The government then increases its spending notably less in response to a positive technology shock, letting the ratio of public to private consumption decline under positive hiring costs (whereas it would stay constant in the benchmark case). This muted response of government spending is consistent with limited (costly) labor reallocations and thus reduced profits. We also find that shocks to the disutility of labor generally have effects of the opposite sign than technology shocks.

The rest of the paper is organized as follows. We present the model in section 2. The Ramsey policy problem is set up in section 3. We then study the properties of the optimal steady state allocation both analytically and numerically in section 4. Section 5 then discusses the model economy's responses to technology and preference shocks, whereupon section 6 concludes.

2 The model

We analyze a dynamic general equilibrium model in which households accumulate capital and supply labor. Fiscal policy decides on the level of government spending, which provides utility to households, and the level of a proportional labor income tax rate. The labor market part is modelled as in Blanchard and Gali (2010), in particular, hiring of labor by firms is costly. Our model differs from theirs in that we also consider capital accumulation and, most importantly, concentrate on fiscal, not monetary policies, assuming flexible prices.[1]

Let s_t denote the state realized at date $t \geq 0$ and let s^t denote a particular history of states from period 0 to t, $s^t = \{s_t, s_{t-1}, ..., s_0\}$, where $S : s_t \in S$ is the set of possible states and S^t the set of possible histories. Further, let $\pi(s^t|s^{t-1})$ be the period $(t-1)$ probability of the occurrence of the history s^t, and $\pi(s^t) = \pi(s^t|s_0)$ its unconditional probability, where s_0 is the initial state with $\pi(s_0) = 1$. Throughout the paper, we economize on notation by leaving out the reference to the state s^t wherever possible without risk of confusion.

2.1 Households

There is a representative household consisting of a continuum of members (indexed with j), normalized to measure one. The household members' preferences are identical, but members may differ with regard to their employment status. Let $n_t \in (0,1)$ denote the fraction of employed household members. These work the fixed amount of one time unit per period and earn a real wage w_t. Employment reduces leisure and utility by a fixed amount $\chi > 0$. Expected lifetime utility of the household is then given by $\sum_{t=0}^{\infty} \sum_{s^t \in S^t} \pi\left(s^t\right) \beta^t \left\{ \int_0^1 \left[u\left(c_{jt}\left(s^t\right)\right) + v(g_t\left(s^t\right))\right] dj - \int_0^{n\left(s^t\right)} \chi dj \right\}$, with $u(.)$ and $v(.)$ logarithmic in consumption and government spending, respectively, which can be rewritten as

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \int_0^1 \left[\ln c_{jt} + \varphi \ln g_t\right] dj - \int_0^{n_t} \chi_t dj \right\}, \qquad \beta \in (0,1), \tag{1}$$

where $\varphi > 0$ is a constant utility parameter that determines the relative valuation of public and private consumption and E_0 is the expectations operator contingent on the information in period 0, c_{jt} is private consumption of the j-th member, and g_t is government spending which produces nonrival public goods that are consumed by all household members in the same amount. The disutility of labor, χ_t, is assumed to follow the exogenous shock process $\chi_t = \rho_d \chi_{t-1} + (1-\rho_d)\chi + \varepsilon_t^d$, with $\rho_d \in (0,1)$, $E_t \varepsilon_{t+1}^d = 0$, and a constant steady

[1] An earlier version of the paper (see Linnemann and Schabert, 2009) also considered jointly optimal fiscal and monetary policies under price stickiness.

state level $\chi > 0$. We refer to exogenous variations in χ_t as preference shocks, henceforth.

All household members have access to a perfect capital market where a complete set of one period contingent claims is traded. The reason for assuming perfect capital markets is to avoid differences in asset holdings and consumption levels among household members; thus, heterogeneity refers to the employment status only (like in Merz, 1995, and a large subsequent literature).

In each period t household members trade claims to period $t+1$, whose payment is contingent on the realization of s_{t+1}. Let $Q_{t,t+1}(s^t, s_{t+1})$ be the period-t-price of one unit of the consumption good in a particular state s^{t+1} in period $t+1$. When the j-th household member's portfolio of state contingent claims yields a random payment $m_{jt+1}(s^t, s_{t+1})$ in period $t+1$, then the period t price of a random payoff is given by $\sum_{s_{t+1}\in S} Q_{t,t+1}(s^t, s_{t+1}) m_{jt+1}(s^t, s_{t+1}) = E_t[\phi_{t,t+1} m_{jt+1}]$, where $m_{jt+1} = m_{jt+1}(s^t, s_{t+1})$ and $\phi_{t,t+1} = Q_{t,t+1}(s^t, s_{t+1})/\pi(s^{t+1}|s^t)$ is the stochastic discount factor.

Households can further invest in physical capital and receive profits q_{jt} from firms, which they own (share holdings are not explicitly modelled, for simplicity). The household's flow budget constraint can then be written as

$$\int_0^1 E_t[\phi_{t,t+1} m_{jt+1}] dj \leq \int_0^1 (m_{jt} - c_{jt} - i_{jt} + r_t k_{jt-1})\, dj + (1-\tau_t) \int_0^{n_t} w_t dj + q_{jt}, \qquad (2)$$

where i_{jt} is investment in physical capital k_{jt}, earning a real return r_t when lent out to the firm sector for one period, and τ_t denotes a flat rate labor income tax rate. Lump-sum taxes are not available, such that the government is forced to finance its expenditure through distortionary labor taxation and debt. Capital is accumulated according to

$$k_{jt} = (1-\delta) k_{jt-1} + i_{jt}, \qquad (3)$$

where $\delta \in (0,1)$ is a fixed depreciation rate. Maximizing expected lifetime utility of the household subject to the budget constraint (2) and (3), as well as to a no-Ponzi-game condition $\lim_{t\to\infty} E_0[\phi_{0,t} m_{jt+1}] \geq 0$ and a non-negativity condition for capital for given initial values $k_{j,-1}$ and m_{j0}, leads to the following first order conditions for consumption and investment in contingent claims and in physical capital[2]:

$$c_{jt}^{-1} = \Lambda_t,$$

$\phi_{t,t+1} = \beta \frac{\Lambda_{t+1}}{\Lambda_t}$, $\Lambda_t = \beta E_t \Lambda_{t+1} \left[r_{t+1} + 1 - \delta \right]$, and transversality conditions $\lim_{t\to\infty} \beta^t E_0 \Lambda_t k_{jt} = 0$ and $\lim_{t\to\infty} \beta^t E_0 \Lambda_t m_{jt+1} = 0$, where Λ_t is the multiplier on the flow budget (2). As a consequence, all household members $h \neq j$ exhibit an identical marginal utility of con-

[2] We assume that all agents perceive the law of motion for the aggregate state to follow a first order Markov process.

sumption $c_{jt}^{-1} = c_{ht}^{-1} = \Lambda_t$, and we can rewrite the first order conditions in terms of aggregate household consumption,

$$\phi_{t,t+1} = \beta \frac{c_{t+1}^{-1}}{c_t^{-1}}, \quad (4)$$

$$c_t^{-1} = \beta E_t c_{t+1}^{-1} \left[r_{t+1} + 1 - \delta \right], \quad (5)$$

where c_t denotes aggregate household consumption satisfying $c_t = \int_0^1 c_{jt} dj$ and $c_t = c_{jt}$ for all j. Furthermore the transversality conditions $\lim_{t\to\infty} E_0 \beta^t c_t^{-1} k_t = 0$ and $\lim_{t\to\infty} E_0 \beta^t c_t^{-1} m_{t+1} = 0$, where $m_t = \int_0^1 m_{jt} dj$ and $k_t = \int_0^1 k_{jt} dj$, have to hold.

2.2 Firms

There is a continuum of perfectly competitive firms index with i, where $i \in [0,1]$. The i-th firm's technology is

$$y_{it} = a_t n_{it}^v k_{it-1}^{1-v}, \qquad v \in (0,1],$$

where a_t is the common stochastic level of productivity satisfying $a_t = \rho_a a_{t-1} + (1-\rho_a) a + \varepsilon_t^a$, $\rho_a \in (0,1)$, where $E_{t-1} \varepsilon_t^a = 0$ holds for the innovation, and the constant is $a = 1$.

Following Blanchard and Gali (2010), firms can hire instantaneously, but hires h_{it} are associated with costs z_t per hire (assumed to be the same for all firms). The costs per hire, which are taken as given by each individual firm, are assumed to be increasing and convex in the level of aggregate labor market tightness x_t,

$$z_t = b x_t^{\alpha}, \qquad \alpha, b > 0,$$

where tightness is defined as the ratio of aggregate hires $h_t = \int h_{it} di$ to the number of unemployed at the beginning of the period u_t,

$$x_t = h_t / u_t \in [0,1].$$

Thus, x_t can be interpreted as a job finding rate. Further, allowing for separation in each period, the number of worker in each firm evolves according to

$$n_{it} = (1-d) n_{it-1} + h_{it}, \quad (6)$$

where $d \in (0,1)$ is the exogenous separation rate. Thus, the total amount of hires satisfies:

$$h_t = n_t - (1-d) n_{t-1}, \quad (7)$$

where $n_t = \int n_{it} di$ denotes the number of employed. Accordingly, the beginning-of-period (before hiring) measure of unemployment u_t satisfies: $u_t = 1 - (1-d) n_{t-1}$, while unemployment after hiring is given by $1 - n_t$.

The i-th firm is assumed to maximize the following expected discounted sum of real period profits q_{it},

$$\max E_t \sum_{s=0}^{\infty} \phi_{t,t+s} q_{it+s}$$

$$\text{with } q_{it+s} = \frac{P_{it+s} y_{it+s} - P_{t+s} w_{t+s} n_{it+s} - P_{t+s} r_{t+s} k_{it+s-1} - P_{t+s} z_{t+s} h_{it+s}}{P_{t+s}},$$

where the firm applies the owners' stochastic discount factor $\phi_{t,t+s} = \beta^s \frac{c_{t+s}^{-1}}{c_t^{-1}}$, subject to $y_{it+s} = a_{t+s} n_{it+s}^{v} k_{it+s-1}^{1-v}$ and $h_{it+s} = n_{it+s} - (1-d) n_{it+s-1}$, taking the real wage and capital rental rate and firm-level costs per hire z_{t+s} as well as previous period employment and capital $n_{i,-1}$ and $k_{i,-1}$ as given.

Letting Θ_{it} be the multiplier on (6), the first order conditions read

$$w_t = v a_t n_{it}^{v-1} k_{it-1}^{1-v} - \Theta_{it} + (1-d) E_t \beta \frac{c_{t+1}^{-1}}{c_t^{-1}} \Theta_{t+1}, \tag{8}$$

$$\Theta_{it} = z_t, \tag{9}$$

$$r_t = (1-v) a_t n_{it}^{v} k_{it-1}^{-v}. \tag{10}$$

Note that Θ_{it} gives the marginal value of an additional hire to the firm, and is equal to the (per-hire) hiring costs $z_t = b x_t^{\alpha}$ that are saved when an additional worker is in place; this is the same across firms, and therefore the marginal value of a hire is the same across firms, implying $\Theta_{it} = \Theta_t$. Note that there exists an externality, since an individual firm does not take into account the impact of its own hires h_{it} on the hiring costs z_t.

$$z_t = v a_t n_{it}^{v-1} k_{it-1}^{1-v} - w_t + (1-d) E_t \beta \frac{c_{t+1}^{-1}}{c_t^{-1}} z_{t+1} \tag{11}$$

2.3 Wage bargaining

Following Blanchard and Gali (2010) as well as a large part of the macroeconomic literature on unemployment, it is assumed that real wages are determined in a Nash bargain between workers and firms. Let ω_t^n be the household's value of being employed, and ω_t^u the value of being unemployed at the beginning of t. We have

$$\omega_t^n = (1-\tau_t) w_t - \chi_t c_t + \beta E_t \frac{c_{t+1}^{-1}}{c_t^{-1}} \left[d(1-x_{t+1}) \omega_{t+1}^u + (1 - d(1-x_{t+1})) \omega_{t+1}^n \right].$$

Here $d(1-x_{t+1})$ is the probability of the transition from employed to unemployed status (d is the separation rate, and x_{t+1} the job finding rate for period $t+1$, such that $d(1-x_{t+1})$ is the probability of being fired and then not finding a job next period). The value of being unemployed is

$$\omega_t^u = \beta E_t \frac{c_{t+1}^{-1}}{c_t^{-1}} \left[(1-x_{t+1})\omega_{t+1}^u + x_{t+1}\omega_{t+1}^n\right].$$

The household's surplus in the Nash bargain is thus

$$\omega_t^n - \omega_t^u = (1-\tau_t)\, w_t - \chi c_t + \beta(1-d)E_t \frac{c_{t+1}^{-1}}{c_t^{-1}} \left[(1-x_{t+1})\left(\omega_{t+1}^n - \omega_{t+1}^u\right)\right]. \tag{12}$$

The surplus of a firm i's hire Θ_{it} is equal to the (per-hire) hiring costs $z_t = bx_t^\alpha$ that are saved when an additional worker is in place. Given that the costs per hire are identical for all firms, $\Theta_{it} = \Theta_t$, firms and households maximize the Nash product

$$\left(\omega_t^n - \omega_t^u\right)^\kappa \left(\Theta_t\right)^{1-\kappa}, \qquad \kappa \in (0,1),$$

leading to the first order condition

$$\omega_t^n - \omega_t^u = (1-\tau_t)\, \vartheta b x_t^\alpha,$$

where $\vartheta = \frac{\kappa}{1-\kappa}$ is the workers' relative bargaining weight. Inserting this into (12) shows that the bargained real wage satisfies

$$w_t = \frac{\chi_t c_t}{1-\tau_t} + \vartheta b x_t^\alpha - \beta(1-d)E_t \frac{c_{t+1}^{-1}}{c_t^{-1}} \left[(1-x_{t+1})\frac{1-\tau_{t+1}}{1-\tau_t}\vartheta b x_{t+1}^\alpha\right]. \tag{13}$$

2.4 Government

The government levies proportional taxes at the rate τ_t on labor income for purchases of goods $g_t \geq 0$, and can borrow and lend in terms of state contingent claims m_t^g. The government's flow budget identity is thus

$$g_t = \tau_t w_t n_t - E_t[\phi_{t,t+1} m_{t+1}^g] + m_t^g. \tag{14}$$

given $m_0^g \leq 0$. The assumption that the government trades in terms of state contingent claims is made to facilitate the derivation of an intertemporal budget constraint. For a more realistic specification, one might assume that the government combines state contingent claims to a portfolio that resembles a one period bond.

2.5 Rational expectations equilibrium

We consider a symmetric equilibrium, where firms' choices $y_{it} = y_t$, $n_{it} = n_t$ and $k_{it} = k_t$. Hence, their equilibrium behavior can be summarized in terms of aggregate variables only by (7), (10), $y_t = a_t n_t^v k_{t-1}^{1-v}$,

$$
\begin{aligned}
w_t &= v a_t n_t^{v-1} k_{t-1}^{1-v} - b x_t^\alpha + b(1-d) E_t \beta \frac{c_{t+1}^{-1}}{c_t^{-1}} x_{t+1}^\alpha, && (15)\\
q_t &= y_t - w_t n_t - r_t k_{t-1} - z_t h_t, && (16)
\end{aligned}
$$

where we used (11) for (15). It should be noted that firms' profits are in general not zero even though the production function exhibits constant return to scale. This can easily be seen from combining (15)-(16), which leads to

$$
q_t = (1-d) \left\{ n_{t-1} z_t - n_t E_t \beta \left[\frac{c_{t+1}^{-1}}{c_t^{-1}} z_{t+1} \right] \right\}. \qquad (17)
$$

Hence, firms' profits exist since at any point in time the non-separated part of the previous period's employment level is given, which saves the amount of hires and the associated hiring costs. When firms consider their optimal labor demand, they take into account that hires today, to the extent that they will not be separated until the next period, tend to reduce future hiring costs (see 8). Yet, these future expected savings are in general not identical to those arising out of the current period's existing employment level (see 17). Moreover, the average value of profits is strictly positive (even if employment, and thus labor market tightness and hiring costs, do not change) due to discounting, implying that there are pure rents in the long-run (see 17). We will show below that these profits will be relevant for optimal fiscal policy.

In a rational expectations equilibrium markets clear, prices adjust in accordance with the plans of households and firms, and, in particular, the bargained real wage will satisfy (13) and the firms' labor demand condition (15). A rational expectations equilibrium then is a set of sequences $\{n_t, k_t, c_t, x_t, w_t, r_t, q_t, \phi_{t,t+1}, m_t^g\}_{t=0}^\infty$ satisfying (4), (5), (10), (14), (15), (16),

$$
\begin{aligned}
a_t n_t^v k_{t-1}^{1-v} &= c_t + b x_t^\alpha \left[n_t - (1-d) n_{t-1} \right] + g_t + k_t - (1-\delta) k_{t-1}, && (18)\\
\frac{\chi_t c_t}{1-\tau_t} &= v a_t n_t^{v-1} k_{t-1}^{1-v} - (1+\vartheta) b x_t^\alpha && (19)\\
&\quad + \beta b (1-d) E_t \frac{c_{t+1}^{-1}}{c_t^{-1}} \left(x_{t+1}^\alpha + (1 - x_{t+1}) \frac{1-\tau_{t+1}}{1-\tau_t} \vartheta x_{t+1}^\alpha \right), \\
x_t &= \frac{n_t - (1-d) n_{t-1}}{1 - (1-d) n_{t-1}}, && (20)
\end{aligned}
$$

and the transversality conditions, $\lim_{t\to\infty} E_0 \beta^t c_t^{-1} m_{t+1} = 0$ (where $m_t = -m_t^g$) and $\lim_{t\to\infty} E_0 \beta^t c_t^{-1} k_t = 0$, given a fiscal policy choice of the time paths $\{g_t, \tau_t\}_{t=0}^\infty$, exogenous sequences $\{a_t, \chi_t\}_{t=0}^\infty$, and initial values $k_{-1} > 0$, $n_{-1} > 0$, and $m_0^g \le 0$.

3 Optimal fiscal policy

In this section we describe the problem of the government. We assume the government can credibly commit itself, such that its decisions can be derived from a standard Ramsey problem. The Ramsey planner aims at maximizing household welfare (1), which can be rewritten as

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t + \varphi \ln g_t - \chi_t n_t \right\}, \tag{21}$$

subject to the restrictions imposed by private sector behavior (4), (5), (10), (15), (16), (14), (18)-(20), and the transversality conditions.

Using the period-0 price of one unit of the consumption good in period t for a particular history s^t, $Q_0(s^t) = Q_{t-1,t}(s^{t-1}, s_t)\ Q_{t-2,t-1}(s^{t-2}, s_{t-1})\ ...Q_{0,1}(s^0, s_1)$, and applying the first order condition $\phi_{t,t+1} = \frac{Q_{t,t+1}(s^t, s_{t+1})}{\pi(s^{t+1}|s^t)} = \beta \frac{c_{t+1}^{-1}(s^{t+1})}{c_t^{-1}(s_0)}$, which implies $Q_0(s^t) = \beta^t \pi(s^t) \frac{c_t^{-1}(s^t)}{c_0^{-1}(s_0)}$ the intertemporal government budget constraint can be written as

$$m_0^g = \sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t) \beta^t \frac{c_t^{-1}(s^t)}{c_0^{-1}(s_0)} \left(g_t \left(s^t\right) - \tau_t \left(s^t\right) w_t \left(s^t\right) n_t \left(s^t\right) \right),$$

where we used the transversality condition. Omitting the reference to the state for notational simplicity gives

$$m_0^g c_0^{-1} = E_0 \sum_{t=0}^{\infty} \beta^t c_t^{-1} \left(g_t - \tau_t w_t n_t \right). \tag{22}$$

An optimal fiscal policy is a sequence $\{g_t, \tau_t\}_{t=0}^{\infty}$ that implements a rational expectations equilibrium that yields the highest level of household welfare. Thus, it maximizes (21) subject to (4), (5), (10), (15), (16), (18)-(20), and (22), given k_{-1}, n_{-1}, and m_0^g. The rational expectations equilibrium under an optimal policy is characterized in appendix 8.1. In deriving the policy, we ignore time inconsistency issues and assume that the initial values for the predetermined variables are equal to their values in the deterministic steady state.

4 Steady state

In this section, we analyze the steady state of the model under the optimal fiscal policy. We proceed in two steps: first, we analytically characterize the steady state under optimal policy for a simplified version of the model without capital accumulation. In step two, we numerically analyze the properties of the completely parameterized steady state of the full model.

4.1 Analytical results

In appendix 8.2, we list the full set of conditions for a (deterministic) steady state equilibrium under the optimal policy. Before we turn to the steady state analysis of the original problem, we apply a simplified version of the model in order to disclose how optimal fiscal policy depends on labor market frictions. Specifically, we assume that there is no capital and returns to labor are constant ($v = 1$), and that there is no initial public debt ($m_0 = 0$), which allows to derive several results in an analytical way (here and henceforth, variables without time indices denote constant steady state values).

First, consider the limiting case where no labor market frictions are present, i.e. hiring costs are zero (by setting $b = 0$). In this case, where the labor income tax is the only friction, the optimal steady state ratio of public to private consumption would be $g/c = \varphi$ (as shown in appendix 8.2). This policy equates the marginal utilities of public and private consumption, and would also be chosen under the first best allocation, i.e. if neither distortionary taxes nor labor market frictions existed. Hence, absent hiring costs the Ramsey planner would choose the first best ratio of government spending to private consumption in the steady state, even though the labor supply decision is distorted by the income tax.[3] The steady state tax rate for this case is given by $\tau = \varphi/(1+\varphi)$ and the optimal consumption level by $c = \chi^{-1}(1-\tau)$. It should be noted that these results do not hold in general and depend, in particular, on the utility function. The logarithmic utility function we have chosen is particularly instructive, as it produces the first best choice of public to private consumption, namely φ, if there are tax distortions but no labor market frictions, and thus allows to show how the sole introduction of labor frictions can induce the policy maker to deviate from this choice.

Second, consider the case where labor market frictions exist, i.e. where hiring is costly $z > 0$ (due to $b > 0$). As shown in appendix 8.2, the optimal steady state ratio of public to private consumption is then given by

$$g/c = \varphi + q \cdot \Phi, \tag{23}$$

where steady state profits q are given by

$$q = bn(1-\beta)(1-d)(dn/[1-(1-d)n])^{\alpha} > 0 \tag{24}$$

(from 17), and $\Phi > 0$ is defined as $\Phi(\Gamma, g, c, \tau, x) = \vartheta\Gamma\frac{g}{c^2}\frac{bx^{\alpha}(1-\tau)(d(1-x)+x)}{\frac{c\chi}{1-\tau}+x^{\alpha}b\vartheta(1-x)(1-\beta)(1-d)} > 0$. Thus, under an optimal fiscal policy, the ratio g/c deviates from its first best level φ; in particular, since profits q are positive under frictional labor markets (see 24), the public-private consumption ratio is larger than φ with labor frictions. This choice for the ratio

[3] A corresponding result has been shown by Lansing (1997) in a model with government spending in the production function.

g/c is accompanied by an optimal tax policy, which accords to the well-established public finance principle to confiscate pure rents stemming from firms' profits that are transferred to households. This can be seen from the following condition for the optimal steady state labor income tax rate (see appendix 8.2):

$$\tau = \frac{\varphi + q \cdot \Phi}{1 + \varphi + q \cdot (\Phi - u_c)}, \tag{25}$$

where $u_c = 1/c > 0$. Given that there exist pure rents $q > 0$ stemming from saved hiring costs due to the pre-existing employment level (see also 24), the optimal fiscal policy reacts according to (25) by increasing the tax rate above the level $\varphi/(1+\varphi)$ that would be optimal under frictionless labor markets. Further, the optimal consumption level is reduced by hiring costs z, as can be seen from the steady state version of (19) which reads

$$c = \chi^{-1} (1 - \tau) (1 - z \cdot \Psi), \tag{26}$$

where $\Psi(n) = 1 - \beta (1-d) + \vartheta[1 - \beta (1-d) \frac{1-n}{1-(1-d)n}] > 0$ (again see appendix 8.2). Hence, labor market frictions cause the Ramsey planner to choose a higher steady state tax rate and a higher steady state ratio of public to private consumption compared to the case of frictionless labor markets. These results are summarized in the following proposition.

Proposition 1 *For $v = 1$ and $m_0 = 0$, optimal fiscal policy is characterized by a tax rate and a ratio of public to private consumption in the steady state, which are both higher than in the case where no labor market frictions exist.*

If the government would have access to non-distortionary taxation, the intertemporal government budget constraint would not be a binding constraint for the Ramsey problem. The optimal steady state ratio of public to private consumption would then differ from the first best level φ if and only if there are labor market frictions $b > 0$ and additionally the workers' relative bargaining weight ϑ differs from the elasticity of the hiring cost function α (as shown in a working paper version, Linnemann and Schabert, 2008), which accords to a violation of the well-known Hosios condition.[4]

Here, it is important to note that whether the Hosios condition is satisfied or not does not significantly alter optimal fiscal policy when the government does not have access to non-distortionary taxation (see 23 or 25). The reason is that allocative efficiency is already impeded by distortionary labor taxes, which are positive since the government needs to provide useful public goods through positive government spending. The labor market friction studied here interacts with tax distortions in a way that (constrained) efficiency is unattainable for the Ramsey planner irrespective of the fulfillment of the Hosios condition.

[4] Precisely, under an optimal fiscal policy regime with access to lump-sum taxation, the steady state ratio of public to private consumption satisfies $g/c = \varphi + (\vartheta - \alpha) z \, (g/c) \, / \, \left[(1 - (\alpha+1) b x^{\alpha}) + \chi^{-1} \alpha (1+\vartheta) b x^{\alpha-1}\right]$ (see also Linnemann and Schabert, 2008), such that $g/c = \varphi$ if the Hosios condition is satisfied, $\vartheta = \alpha$.

4.2 Numerical analysis

In what follows, we present results from numerical experiments with a parameterized version of the steady state of the complete model (see appendix 8.2).

4.2.1 Calibration

We choose parameters to characterize a continental European country by identifying steady state values with averages computed from time series observations for Germany, France, and Italy.[5] Unless otherwise noted, data are from the European Commission's Annual Macroeconomic Database AMECO and cover the period from 1960 to 2007, where the initial date is due to data availability and the final date is chosen to exclude the turmoil of the recent financial crisis. We choose the discount factor as $\beta = 0.97$ for a yearly interpretation of the model, implying roughly a three percent real interest rate. The labor share in output (AMECO's average adjusted wage share in GDP at factor cost) is 0.6895. The depreciation rate is computed as the average ratio of gross fixed capital formation over the capital stock, yielding 0.0704. The steady state unemployment rate is taken as the average over the sample from 1980 onwards, thus covering the historical episode since the emergence of the European unemployment problem, with a value of 8.55%. This amounts to choosing the utility parameter χ such that $n = 0.9145$ results in the steady state. The average ratio of government consumption over private consumption is 0.3278 in the sample; we use this as our value for the utility parameter φ. Unlike in the previous section, we also allow for a non-zero steady state debt-to-gdp ratio m/y; data are available for all three countries only from 1990 onwards in the AMECO database, which leads to an average value of 74.8%.[6]

Then, we follow Blanchard and Gali (2010) in setting the curvature of the hiring cost function to $\alpha = 1$. Also, we follow Blanchard and Gali (2010) to adjust the parameter b such that in the calibrated steady state the costs of hiring, $bx^{\alpha}dn$, are a certain percentage $\Delta \equiv bx^{\alpha}dn/(n^{v}k^{1-v})$ of steady state output, which leads to a calibrated value for the constant b; in practice, we specify Δ to be 0.01 as in Blanchard and Gali (2010), such that hiring costs represent one per cent of steady state output. However, as the parameter b is at the same time crucial for the results and not directly empirically observable, we will discuss the consequences of varying it below. In the baseline calibration, we assume that labor's bargaining power is as large as the firms', which amounts to setting relative bargaining power to $\vartheta = 1$; we will however conduct a sensitivity analysis with respect to this parameter below.

Finally, for continental European countries Elsby et al. (2011) report values for the

[5] For the period before German unification in 1990, West German data are used.

[6] The steady state debt-to-gdp ratio under the optimal fiscal policy is a function of the initial debt level. Here, we calibrate the steady state debt-to-gdp ratio in accordance with empirical observations, which can in principle be implemented by an appropriate choice of the initial debt level.

monthly job finding rate which are on average about 7%. Thus, to calibrate the annual steady state job finding rate x, we use $x^{monthly} = 0.07$ and compute $x = \sum_{i=0,\ldots,12}(1 - x^{monthly})^i x^{monthly}$. This gives a value of $x = 0.5814$, which together with the steady state condition $x = dn/(1-(1-d)n)$ and our earlier choice of a steady state employment rate of $n = 0.9145$ implies a value for the annual separation rate of $d = 0.1299$. This parameter choice is certainly subject to considerable empirical uncertainty; we therefore checked the robustness of the results with respect to different values. As it turned out (not reported), changes in d affect the steady state qualitatively in the same way as those in b, which are documented below. We therefore abstain from presenting results for different values of d for brevity. The following table 1 gives an overview over the chosen or calibrated parameter values.

Table 1 Parameter values

β	δ	v	α	ϑ	χ	n	φ	d	b	m/y
0.97	0.0704	0.6895	1	1	0.8447	0.9145	0.3278	0.1299	0.22	0.748

4.2.2 Results

With the parameter value given in table 1, we compute the steady state under optimal policy numerically. This results, for the baseline case, in a steady state tax rate of 0.3156, a ratio of government spending to output of 0.1914, and of consumption to output of 0.5829. The implied ratio of government spending to private consumption is $g/c = 0.3283$, which is slightly larger than the utility parameter $\varphi = 0.3278$. Thus, the steady state result that $g/c > \varphi$ for the case of positive hiring costs that has been established analytically is confirmed numerically for the full version of the model with endogenous capital accumulation and positive steady state public debt. For the chosen calibration, the effect is quantitatively small though.

We now turn to demonstrating the effect of varying key parameters on the steady state optimal policy. In figure 1, the parameter b that governs the size of hiring costs, and thus the severity of labor market frictions, is varied on the horizontal axis. For each value, the resulting steady state values of the variables given in the panel titles are plotted.

Since b determines the quantitative importance of the hiring costs, varying it gives an indication how optimal fiscal policy would respond, in the steady state, to different degrees of the distortion in the labor market. In accordance with proposition 1, optimal policy keeps the steady state ratio of public to private consumption at its first best level $g/c = \varphi = 0.3278$ if and only if there are no hiring costs, $b = 0$. Under positive hiring costs, $b > 0$, the ratio g/c monotonically increases with b, as does the optimal labor tax rate τ. The reason is that, on the one hand, hiring frictions lead to the existence of positive profits, which the planner chooses to tax indirectly. On the other hand, a more costly labor

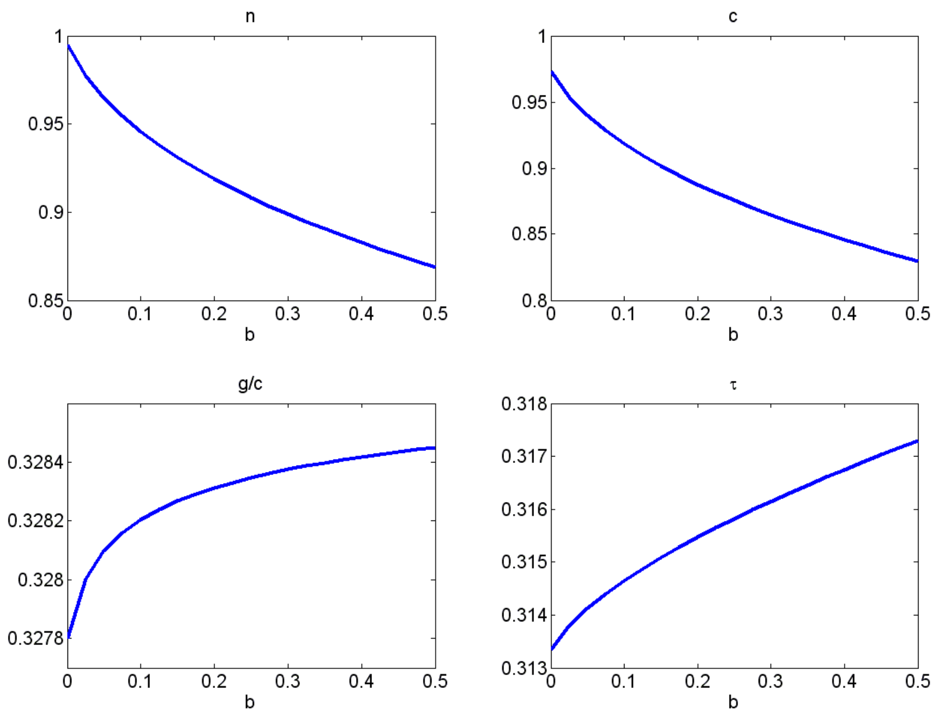

Figure 1: Steady state allocations under optimal policy for varying b.

reallocation process lowers employment and consumption. The planner cannot undo the effects of the hiring costs, since this would entail subsidizing labor by choosing a negative tax rate. This is not possible in the present model, since the government provides a positive amount of public goods that are valued by households, and therefore needs to choose positive tax rates. Since labor reallocation is more costly the higher are hiring costs, employment is lower and the tax rate increases with b which compensates for the reduction in the tax base. As shown analytically for the simplified version of the model above in (26), private consumption is lower the larger are hiring costs, which is exacerbated by the higher tax rate. The planner then chooses to increase utility providing government expenditures relatively to private consumption (although the levels of both decline, government expenditures decline less). Hence, the Ramsey-optimal ratio of government spending to private consumption g/c is higher than its first best level φ, and increases in the size of hiring costs.

Figure 2 shows the effect on the optimal policy in the steady state of varying labor's relative bargaining power ϑ. The effects of higher bargaining power on employment, consumption, and the public-private spending ratio g/c are similar to those of higher hiring costs, but the effect on the tax rate is opposite. Higher ϑ means that a larger part of the surplus from existing hires is transferred to households through the wage bargain, which tends to reduce firms' labor demand. To mitigate the adverse employment and

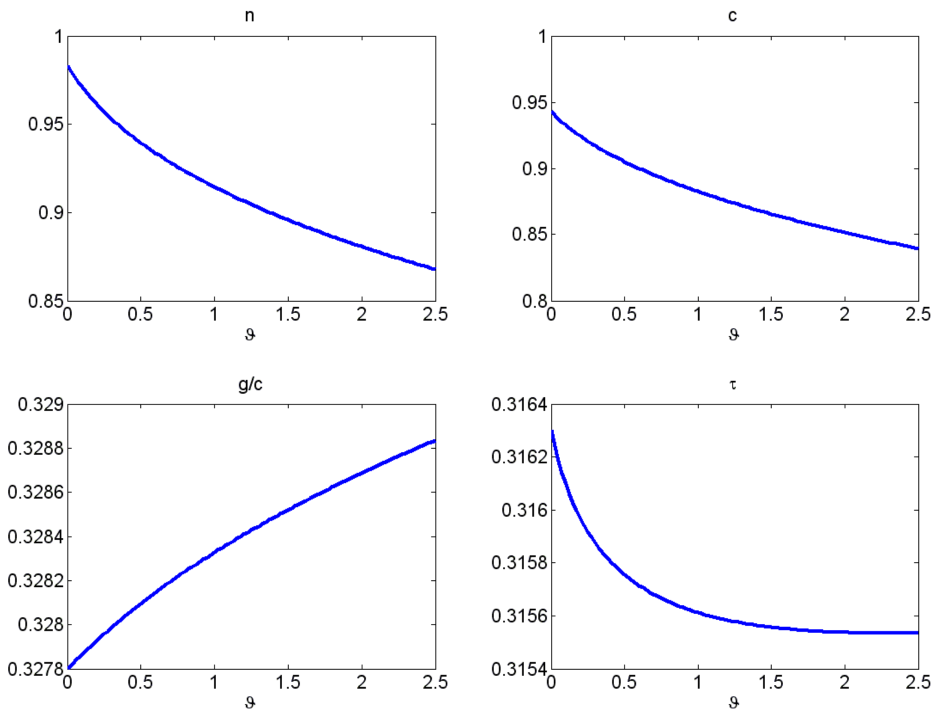

Figure 2: Steady state allocations under optimal policy for varying ϑ.

consumption effects of higher wage claims, the tax rate declines with increasing ϑ.[7] As above, government expenditures increase relatively for higher values of ϑ, which has the effect of limiting the utility loss from declining private consumption.

4.2.3 Welfare comparison

The previous sections have shown that optimal policy leads to a steady state where the ratio of government spending to consumption differs from the benchmark $g/c = \varphi$ that would be optimal in a frictionless model, although the difference turned out to be quantitatively small for the chosen calibrations. In this section, we quantify the welfare effects by conducting the following experiment: the economy starts in a steady state which is characterized by a non-optimal policy. In this initial steady state, government spending is simply set at $g/c = \varphi$, and the tax rate is chosen to balance the budget. The initial steady state thus characterizes a situation where labor market frictions exist, but policy ignores them and follows a simple policy which would be optimal if labor market frictions were absent. The welfare received from staying forever in this non-optimal deterministic steady state (with consumption, government spending, and employment given by $c^0, n^0,$

[7] If ϑ becomes even larger, the fact that labor income is the tax base (which declines through the employment reducing effect of high bargaining power) would force the planner to abstain from further tax rate reductions and necessitates even slight tax rate increases. Again, this is due to the fact that the planner needs to finance a positive amount of utility enhancing government spending through taxing labor.

and g^0) is denoted v_0, defined as

$$v_0 = \sum_{t=0}^{\infty} \beta^t u(c^0, n^0, g^0).$$

Starting from this steady state, the policy once and for all switches to the optimal policy. Let the deterministic time paths of consumption, employment, and government spending resulting from this policy switch be c_t^1, n_t^1, and g_t^1. We compute these time paths as the deterministic transition to the new steady state after the policy switch.[8] Welfare under this scenario (including the phase of transition) is denoted v_1, given as

$$v_1 = \sum_{t=0}^{\infty} \beta^t u(c_t^1, n_t^1, g_t^1).$$

We then compute the consumption equivalent of the welfare consequences of the policy shift. Specifically, we ask how much of the initial steady state consumption the representative household would pay for the shift from the non-optimal to the optimal policy. Let the constants $\overline{c}^0$ and $\overline{c}^1$ be the permanent consumption streams that yield v_0 and v_1, respectively: $\ln\left(\overline{c}^0\right)/(1-\beta) = v_0$ and $\ln\left(\overline{c}^1\right)/(1-\beta) = v_1$. We then measure the consumption equivalent Δ of the welfare gain from the transition to the optimal policy as $\Delta = 100 \cdot \left(1 - \overline{c}^0/\overline{c}^1\right)$.

For the baseline parameters discussed above, this results in $\Delta = 0.0077$. If we do not take into account the transition phase, but just compare steady state welfare differences, the corresponding figure would be larger and equal to 0.0117. Thus, if the government does not take into account the existence of labor market frictions and, instead of optimizing, follows the simple rule of setting $g/c = \varphi$, this results in a welfare loss that, to households, is worth less than one hundredth of a percentage point of steady state consumption (or slightly more than that if not taking the transition to the new steady state into account). In this sense, while $g/c = \varphi$ is not optimal in an economy with labor frictions, the welfare consequences of this non-optimality are very small in this model.

5 Dynamic effects

We now examine the short-run macoreocnomic dynamics, i.e. responses to exogenous shocks. To study the cyclical properties under the optimal policy, we take a loglinear approximation of the equilibrium conditions at the deterministic steady state.[9] We found the steady state to be saddle path stable in all cases considered. Figure 3 shows percentage deviations from steady state to a positive one percent autocorrelated ($\rho = 0.8$) technology shock; the exception is the lower right panel, where firms' profits q are shown in absolute

[8] Computations have been performed using Dynare (see www.dynare.org).

[9] Computations are carried out in Dynare (see www.cepremap.cnrs.fr/dynare).

deviations from their steady state.[10] In the figure, the responses under the optimal policy for the baseline calibration are shown by solid lines; for comparison, the dotted lines show the case of no labor market frictions ($b = 0$). The dashed lines show the first best case, i.e. the absence of both labor market frictions and the availability of lump-sum taxes (this case is obtained from the problem of maximizing household utility subject to the resource constraint with $b = 0$ and lump-sum taxes).

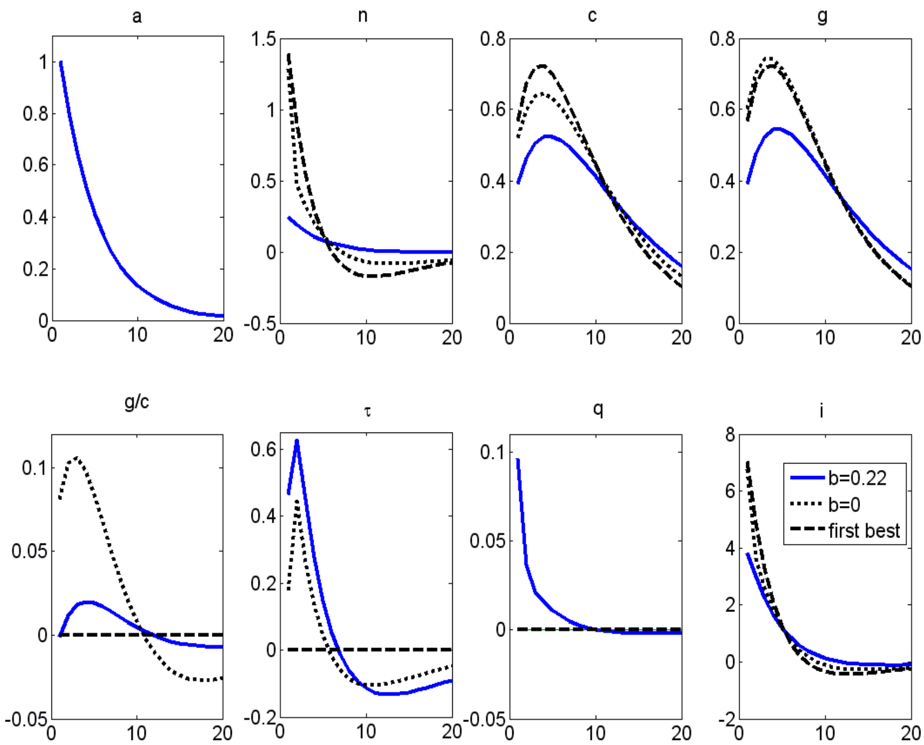

Figure 3: Percentage impulse responses to technology shock, optimal policy

A technology shock always leads to positive responses of all variables, but the quantitative size of the response is different when labor market frictions are present. Consider first the case where labor frictions are absent $b = 0$ (see dotted line). The transitory increase in productivity leads to higher employment and wages, which induces the government to substitute taxation from the future to the present, such that the tax rate is first increased and later decreased. Given that the higher tax rate tends to reduce consumption (see 19 and 23), the increase in consumption is less pronounced than in the first best case (dashed lines). Since both employment and (not shown) the real wage rise, the tax base and thus total tax revenues rise strongly on impact. Accordingly, the share of public to private consumption increases relative to its steady state value during the adjustment process, before

[10] Since the level of profits is very small in the steady state, showing percentage deviations would distort the scale of the figure.

declining below steady state when the employment effects of the shock have vanished and taxes decline. Note that in the first best case the share of public to private consumption is held constant at φ.

With hiring costs ($b > 0$), the employment response is strongly dampened relative to the cases without labor frictions, since labor reallocations are socially costly in this case. As can be seen from the figure, the government increases taxes relatively strongly. It taxes firms' profits which rise in a boom due to the existence of hiring costs such that the tax response is more pronounced than for $b = 0$. In total, the output effect of the technology shock is reduced and consumption as well as government spending and investment expand less than in the frictionless case.

In figure 4, we show the responses that would occur if the tax rate were held constant at its steady state value, while government spending is free to adjust optimally. The latter case seems practically relevant in the short run, since usually tax codes can only be changed through a potentially lengthy parliamentary process, whereas governments typically have some leeway to adjust spending over the business cycle. In this case, the dynamics off steady state under the frictionless case with distortionary taxes (dotted lines) and in the first best case are the same (which is therefore not shown in the figure). When labor market frictions are absent (see dotted line), it is optimal for the government to keep the ratio of public to private spending constant. If however hiring is costly (solid line), it is optimal to let employment increase to a smaller extent to limit labor reallocation costs, which leads to lower tax renevues in the case without hiring costs (even though wages increase). Hence, the optimal policy is to increase government spending much less (compared to the case of variable tax rates seen in figure 3). As a result, the ratio g/c declines if there are labor frictions, whereas it would stay constant in the case of no hiring costs. Thus, the government adjusts uses its spending to smaller employment fluctuations when the tax rate is not at its disposal. In particular, the smaller increase in g in this case is associated with a larger increase in c (correspondingly the notable drop in g/c occurs), which leads to relatively higher wage claims (see **13**) consistent with the muted employment reaction. Hence, the government departs from a policy of keeping the marginal utilities of government spending and private consumption aligned.[11]

Figure 5 shows the impulse responses (for optimally set tax rates and government spending) to an autocorrelated ($\rho = 0.8$) positive shock to the preference parameter χ_t which increases the disutility of working, and induces households to reduce labor supply. Overall, the pattern of the optimal policy responses accords to the responses to a negative productivity shock (see figure **3**). The government reacts by letting the tax rate decline and lowers the ratio of public to private consumption. When hiring is costly (see solid lines), the tax rate reduction is even more pronounced – due to reduction in profits, while the

[11] The mechanism with constant taxes imposed is qualitatively the same as the one under lump-sum taxes that has been discussed in the working paper version, Linnemann and Schabert (2009).

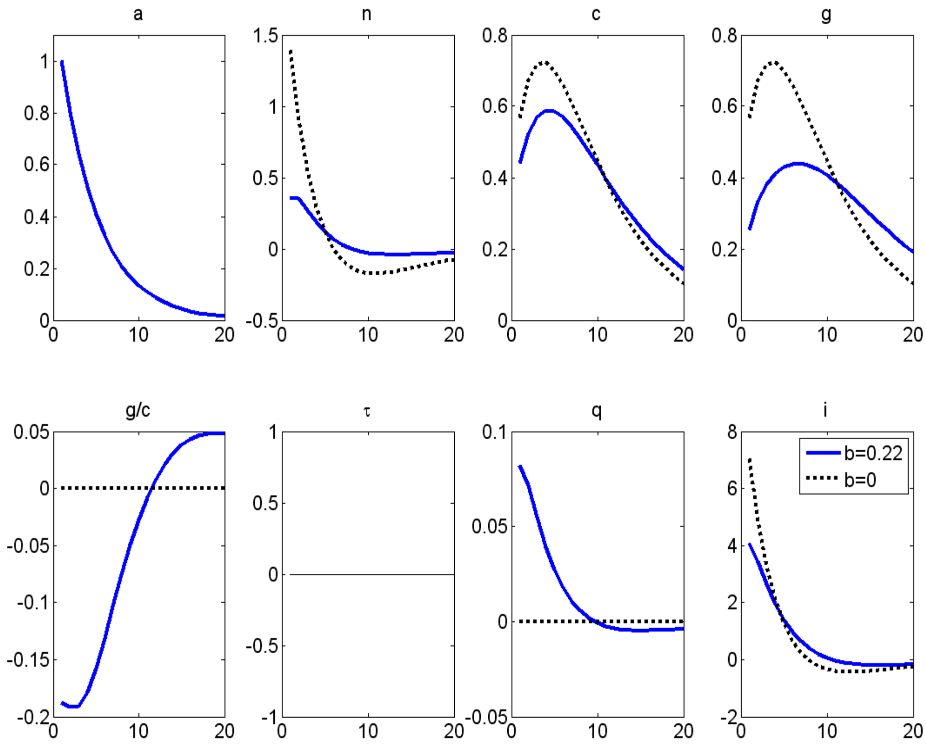

Figure 4: Percentage impulse responses to technology shock, constant tax rate

ratio of public to private consumption responds less than in the case without labor market frictions (see dotted line) and thus limits the labor reallocation costs since it contains the employment decrease. As in the case of technology shocks discussed above, it turns out that the fiscal planner uses strong tax variations and achieves a muted employment response, while aiming to keep government spending and consumption closely linked to each other. If labor reallocation is not costly (dotted lines), employment reacts much more strongly negatively, which reduces the tax base more strongly. Together with a tax reduction, which is weaker due to the absence of profits, government spending decreases more strongly than under positive hiring costs.

6 Conclusion

In this paper, we have analyzed the optimal conduct of government spending and labor income tax policy in a model with frictional unemployment. We use a setting where in the absence of labor market frictions the optimal Ramsey policy would call for choosing steady state government spending so as to equalize their marginal utility to the one of private consumption, such that the ratio of public to private consumption would be determined by the utility function parameter that gives the relative weight of useful public spending

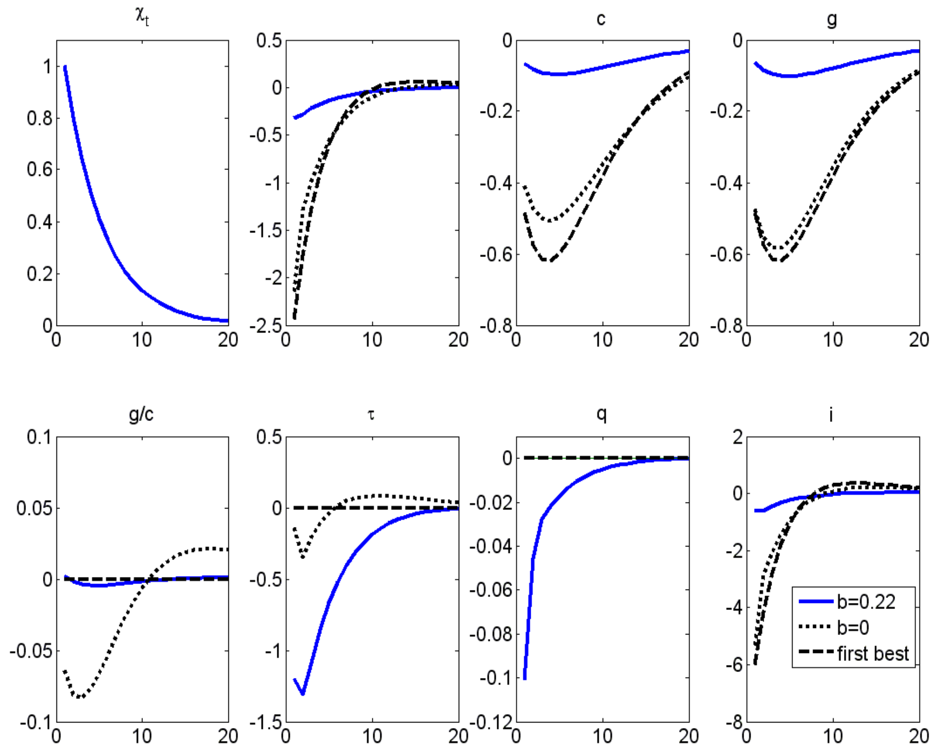

Figure 5: Percentage impulse responses to preference shock, optimal policy

in utility. With labor market frictions, the optimal policy entails a steady state where government spending is higher relatively to private consumption. The quantitative size of this departure is low, however, in a calibrated quantitative model version.

With respect to short-run dynamics, the optimal policy is to use strongly procyclical tax rate adjustments in response to technology or preference shocks. Through this policy, the profits that arise from hiring cost savings of existing employment are taxed indirectly. The ratio of government spending to private consumption reacts only weakly procyclically. However, if the tax rate is assumed to be fixed in the short run, the public to private consumption ratio is markedly countercyclical.

The analysis in the present paper uses a purely real model without any role for nominal variables. Recently, several authors have incorporated labor market frictions in a New Keynesian sticky-price environment (e.g. Blanchard and Gali, 2010, Krause and Lubik, 2007, Gertler and Trigari, 2009, or Faia, 2008) to study optimal monetary policy. The analysis of optimal fiscal policy along the lines suggested in this paper in a setting with both labor frictions and nominal rigidities seems like a natural extension of this literature and is left for future research.

7 References

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8 Appendix

8.1 Optimal fiscal policy

Let Γ be the multiplier on the implementability constraint (22), and let λ_t, ψ_t, μ_t, and ϖ_t be the (time variable) multipliers on the resource constraint (18), on the definition of the job finding rate (20), on the labor market equilibrium condition (19), and on the Euler equation (5) respectively, such that the Lagrangian of the optimal policy problem reads

$$\begin{aligned}
L \;=\; & E_0 \sum_{t=0}^{\infty} \beta^t \{\ln c_t + \varphi \ln g_t - \chi_t n_t \\
& + \Gamma \left[c_t^{-1} \left(-g_t + \tau_t v a_t n_t^v k_{t-1}^{1-v} - \tau_t n_t b x_t^{\alpha} + \tau_t n_t \beta E_t \frac{c_{t+1}^{-1}}{c_t^{-1}} b(1-d) x_{t+1}^{\alpha} \right) \right] \\
& + \lambda_t \left[a_t n_t^v k_{t-1}^{1-v} - c_t - b x_t^{\alpha+1} \left[1 - (1-d) n_{t-1} \right] - g_t - k_t + (1-\delta) k_{t-1} \right] \\
& + \psi_t \left[x_t - \frac{n_t - (1-d) n_{t-1}}{1 - (1-d) n_{t-1}} \right] \\
& + \mu_t \left[-\frac{\chi_t c_t}{1-\tau_t} + v a_t n_t^{v-1} k_{t-1}^{1-v} - (1+\vartheta) b x_t^{\alpha} \right. \\
& \qquad \left. + \beta b (1-d) \frac{c_{t+1}^{-1}}{c_t^{-1}} \left[x_{t+1}^{\alpha} + (1 - x_{t+1}) \frac{1-\tau_{t+1}}{1-\tau_t} \vartheta x_{t+1}^{\alpha} \right] \right] \\
& + \varpi_t \left[\beta E_t \frac{c_{t+1}^{-1}}{c_t^{-1}} \left[(1-v) a_{t+1} n_{t+1}^v k_t^{-v} + 1 - \delta \right] - 1 \right] \Bigg\} \\
& - \Gamma A_0.
\end{aligned}$$

where we have eliminated the wage rate in the implementability constraint by the labor demand condition (15).

The optimal policy then has to satisfy the following first order conditions with respect to the optimal choice of government spending, the tax rate, consumption, employment, the job finding rate, and capital for $t \geq 1$:

$$\frac{\partial L}{\partial g_t} = 0 = \varphi g_t^{-1} - \lambda_t - \Gamma c_t^{-1},$$

$$\begin{aligned}
\frac{\partial L}{\partial k_t} \;=\; & 0 = \Gamma \beta E_t c_{t+1}^{-1} \tau_{t+1} v (1-v) a_{t+1} n_{t+1}^v k_t^{-v} \\
& - \lambda_t + \beta E_t \lambda_{t+1} \left[(1-v) a_{t+1} n_{t+1}^v k_t^{-v} + 1 - \delta \right] + \beta E_t \mu_{t+1} v (1-v) a_{t+1} n_{t+1}^{v-1} k_t^{-v}, \\
& - \varpi_t \beta E_t \frac{c_{t+1}^{-1}}{c_t^{-1}} v (1-v) a_{t+1} n_{t+1}^v k_t^{-v-1},
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial L}{\partial c_t} &= 0 = c_t^{-1} \\
&-\Gamma c_t^{-2}\left(-g_t + \tau_t v a_t n_t^v k_{t-1}^{1-v} - \tau_t n_t b x_t^\alpha\right) - \Gamma c_t^{-2}\tau_{t-1} n_{t-1} b(1-d) x_t^\alpha \\
&-\lambda_t - \mu_t \frac{\chi_t}{1-\tau_t} + \mu_t \frac{1}{c_t}\beta(1-d) b E_t \frac{c_t}{c_{t+1}}\left[x_{t+1}^\alpha + (1-x_{t+1})\frac{1-\tau_{t+1}}{1-\tau_t}\vartheta x_{t+1}^\alpha\right] \\
&-\mu_{t-1}\frac{1}{c_t}(1-d) b \frac{c_{t-1}}{c_t}\left[x_t^\alpha + (1-x_t)\frac{1-\tau_t}{1-\tau_{t-1}}\vartheta x_t^\alpha\right] \\
&+\varpi_t\left[\beta E_t \frac{1}{c_t}\frac{c_t}{c_{t+1}}\left[(1-v)\, a_{t+1} n_{t+1}^v k_t^{-v} + 1 - \delta\right] - 1\right] \\
&-\varpi_{t-1}\left[\frac{1}{c_t}\frac{c_{t-1}}{c_t}\left[(1-v)\, a_t n_t^v k_{t-1}^{-v} + 1 - \delta\right] - 1\right],
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial n_t} &= 0 = -\chi_t \\
&+\Gamma c_t^{-1}\tau_t v^2 a_t n_t^{v-1} k_{t-1}^{1-v} - \Gamma c_t^{-1}\tau_t b x_t^\alpha + \Gamma E_t c_{t+1}^{-1}\tau_t \beta b(1-d) x_{t+1}^\alpha \\
&+\lambda_t v a_t n_t^{v-1} k_{t-1}^{1-v} - \psi_t \frac{1}{1-(1-d) n_{t-1}} \\
&+\beta E_t \lambda_{t+1} b x_{t+1}^{\alpha+1}(1-d) + \beta E_t \psi_{t+1}\frac{(1-d)\,(1-x_{t+1})}{1-(1-d) n_t} \\
&+\mu_t v(v-1) a_t n_t^{v-2} k_{t-1}^{1-v}, \\
&+\varpi_{t-1}\frac{c_t^{-1}}{c_{t-1}^{-1}} v\,(1-v)\, a_t n_t^{v-1} k_t^{-v},
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial x_t} &= 0 = -\Gamma c_t^{-1} b\alpha x_t^{\alpha-1} + \Gamma c_t^{-1}\tau_{t-1} n_{t-1} b(1-d)\alpha x_t^{\alpha-1} \\
&-(\alpha+1)\lambda_t b x_t^\alpha\left[1-(1-d) n_{t-1}\right] + \psi_t - \mu_t \alpha(1+\vartheta) b x_t^{\alpha-1} \\
&+\mu_{t-1} b(1-d)\frac{c_{t-1}}{c_t} x_t^{\alpha-1}\left(\alpha\,(1+\vartheta) - (1+\alpha)\,\vartheta\frac{1-\tau_t}{1-\tau_{t-1}} x_t\right),
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial L}{\partial \tau_t} &= 0 = \Gamma c_t^{-1}\left(v a_t n_t^v k_{t-1}^{1-v} - b n_t x_t^\alpha + n_t \beta E_t \frac{c_{t+1}^{-1}}{c_t^{-1}} b(1-d) x_{t+1}^\alpha\right) \\
&-\mu_t \frac{\chi_t c_t}{(1-\tau_t)^2} \\
&+\mu_t \beta b(1-d)\frac{c_t}{c_{t+1}}(1-x_{t+1})\frac{1-\tau_{t+1}}{(1-\tau_t)^2}\vartheta x_{t+1}^\alpha \\
&-\mu_{t-1} b(1-d)\frac{c_{t-1}}{c_t}(1-x_t)\frac{1}{1-\tau_{t-1}}\vartheta x_t^\alpha,
\end{aligned}
$$

together with (18), (19), (5), (20), and (22).

8.2 Deterministic steady state under the optimal policy

Set of equilibrium conditions In what follows, we summarize the properties which the steady state has to satisfy and compute the steady state values numerically. Constant steady state values of variables are denoted by dropping the time subindex. The steady state values of the stochastic variables are $a = 1$ and $\chi > 0$, respectively. From the policy maker's first order conditions and the constraints, the following 11 conditions for $\{c, g, n, k, x, \tau, \Gamma, \lambda, \psi, \mu, \varpi\}$ characterize the steady state:

$$
\begin{aligned}
0 &= \Gamma c^{-1}\left(vn^{v}k^{1-v} - bnx^{\alpha} + n\beta b(1-d)x^{\alpha}\right) - \mu\frac{\chi c}{(1-\tau)^{2}} \\
&\quad + (\beta-1)\,\mu b(1-d)(1-x)\frac{1}{1-\tau}\vartheta x^{\alpha} \\
0 &= \varphi g^{-1} - \lambda - \Gamma c^{-1} \\
0 &= c^{-1} - \Gamma c^{-2}\left(-g + \tau vn^{v}k^{1-v} - \tau nbx^{\alpha}\right) - \Gamma c^{-2}\tau nb(1-d)x^{\alpha} - \lambda \\
&\quad -\mu\frac{\chi}{1-\tau} + (\beta-1)\,\mu\frac{1}{c}(1-d)b\left[x^{\alpha} + (1-x)\vartheta x^{\alpha}\right] \\
&\quad -\varpi(1-\beta)\frac{1}{c}\left[(1-v)\,n^{v}k^{-v} + 1 - \delta\right] \\
0 &= -\chi + \Gamma c^{-1}\tau v^{2}n^{v-1}k^{1-v} - \Gamma c^{-1}\tau bx^{\alpha} + \Gamma c^{-1}\tau\beta b(1-d)x^{\alpha} \\
&\quad +\lambda vn^{v-1}k^{1-v} - \psi\frac{1}{1-(1-d)n} + \beta\lambda bx^{\alpha+1}(1-d) \\
&\quad +\beta\psi\frac{(1-d)\,(1-x)}{1-(1-d)n} + \mu v(v-1)n^{v-2}k^{1-v} + \varpi v\,(1-v)\,n^{v-1}k^{-v} \\
0 &= -\Gamma c^{-1}b\alpha x^{\alpha-1} + \Gamma c^{-1}\tau nb(1-d)\alpha x^{\alpha-1} - (\alpha+1)\lambda bx^{\alpha}\left[1-(1-d)n\right] + \psi \\
&\quad -\mu\alpha(1+\vartheta)bx^{\alpha-1} + \mu b(1-d)x^{\alpha-1}\left(\alpha\,(1+\vartheta) - (1+\alpha)\,\vartheta x\right) \\
0 &= \Gamma\beta c^{-1}\tau v\,(1-v)\,n^{v}k^{-v} - \lambda + \beta\lambda\left[(1-v)n^{v}k^{-v} + 1 - \delta\right] \\
&\quad +\beta\mu v(1-v)n^{v-1}k^{-v} - \varpi\beta\,(1-v)\,vn^{v}k^{-v-1} \\
(1-\beta)m &= \tau vn^{v}k^{1-v} - \tau nbx^{\alpha}\left[1-\beta(1-d)\right] - g \\
n^{v}k^{1-v} &= c + bx^{\alpha+1}\left[1-(1-d)n\right] + g + \delta k \\
x &= \frac{dn}{1-(1-d)n} \\
\frac{\chi c}{1-\tau} &= vn^{v-1}k^{1-v} - (1+\vartheta)bx^{\alpha} + \beta b(1-d)\left[x^{\alpha} + (1-x)\vartheta x^{\alpha}\right] \\
1/\beta &= (1-v)\,n^{v}k^{-v} + 1 - \delta
\end{aligned}
$$

for a given value $m > 0$, which depends on the initial condition $m_0 > 0$.

Steady state of the simplified version This section derives the results discussed in section 4.1. For a closed form analysis of how an optimal fiscal policy accounts for the labor market friction in the steady state, we introduce some simplifying assumptions. Specifically, we assume here that there is no capital and returns to labor are constant ($v = 1$) and that there is no steady state debt $m = 0$. The set of steady state conditions for $\{c, g, n, x, \tau, \Gamma, \lambda, \psi, \mu\}$ is then given by

$$
\begin{aligned}
0 &= \Gamma c^{-1}\left(n - bnx^{\alpha}\left[1-\beta(1-d)\right]\right) - \mu\frac{\chi c}{(1-\tau)^2} - \mu\left(1-\beta\right) b(1-d)(1-x)\frac{1}{1-\tau}\vartheta & (27)\\
\lambda &= \varphi g^{-1} - \Gamma c^{-1} & (28)\\
0 &= c^{-1} - \Gamma c^{-2}\left(-g + \tau n - \tau n b x^{\alpha}\left[1-(1-d)\right]\right) - \lambda - \mu\frac{\chi}{1-\tau} & (29)\\
&\quad -\mu\left(1-\beta\right)\frac{1}{c}(1-d)bx^{\alpha}\left[1+(1-x)\vartheta\right] & \\
g &= \tau n - \tau n b x^{\alpha}\left[1-\beta(1-d)\right] & (30)\\
n &= c + bx^{\alpha} dn + g & (31)\\
x &= \frac{dn}{1-(1-d)n} & (32)\\
\chi c &= (1-\tau)\left(1 - bx^{\alpha}\left(1-\beta\left(1-d\right)+\vartheta\left(1-\beta\left(1-d\right)\left(1-x\right)\right)\right)\right) & (33)\\
0 &= -\chi + \Gamma c^{-1}\tau - \Gamma c^{-1}\tau b x^{\alpha} + \Gamma c^{-1}\tau\beta b(1-d)x^{\alpha} & (34)\\
&\quad +\lambda - \psi\frac{1}{1-(1-d)n} + \beta\lambda b x^{\alpha+1}(1-d) + \beta\psi\frac{(1-d)\left(1-x\right)}{1-(1-d)n} & \\
0 &= -\Gamma c^{-1} b\alpha x^{\alpha-1} + \Gamma c^{-1}\tau n b(1-d)\alpha x^{\alpha-1} - (\alpha+1)\lambda b x^{\alpha}\left[1-(1-d)n\right] + \psi & (35)\\
&\quad -\mu\alpha(1+\vartheta)bx^{\alpha-1} + \mu b(1-d)x^{\alpha-1}\left(\alpha\left(1+\vartheta\right) - \left(1+\alpha\right)\vartheta x\right) &
\end{aligned}
$$

To identify how optimal fiscal policy depends on the labor market frictions, we apply the steady state conditions (27)-(35). We use (30) to rewrite (27) as

$$\mu\frac{c}{1-\tau}\left(\frac{\chi}{1-\tau} + \left(1-\beta\right) bx^{\alpha}\frac{1}{c}(1-d)(1-x)\vartheta\right) = \Gamma\frac{c^{-1}}{\tau}g \tag{36}$$

and (29) as

$$\mu\left(\frac{\chi}{1-\tau} + \left(1-\beta\right) bx^{\alpha}\frac{1}{c}(1-d)\left(1+(1-x)\vartheta\right)\right) = c^{-1} - \Gamma c^{-2}x^{\alpha}bn\tau\left(1-\beta\right)\left(1-d\right) - \lambda \tag{37}$$

We then divide (37) by (36), to get

$$
\begin{aligned}
\frac{1-\tau}{c}\kappa &= \frac{1 - \varphi\frac{c}{g} - \Gamma c^{-1}x^{\alpha}bn\tau\left(1-\beta\right)\left(1-d\right) + \Gamma}{\Gamma\frac{g}{\tau}} \\
\Leftrightarrow\quad \frac{c}{g}\left(\frac{g}{c} - \varphi\right) &= \Gamma\left(\kappa\frac{1-\tau}{\tau}\frac{g}{c} - 1\right) + \Gamma c^{-1}x^{\alpha}bn\tau\left(1-\beta\right)\left(1-d\right) \qquad (38)
\end{aligned}
$$

where $\kappa = \frac{\frac{\chi}{1-\tau} + (1-\beta)bx^{\alpha}\frac{1}{c}(1-d)(1+(1-x)\vartheta)}{\frac{\chi}{1-\tau} + (1-\beta)bx^{\alpha}\frac{1}{c}(1-d)(1-x)\vartheta} > 1$. Eliminating n in (30) by (31), $\frac{1-\tau}{\tau}\frac{g}{c} = 1 - \left(1-\beta\right)\left(1-d\right)bx^{\alpha}\frac{n}{c}$, and eliminating $\frac{1-\tau}{\tau}\frac{g}{c}$ in (38) then gives

$$
\begin{aligned}
\frac{c}{g}\left(\frac{g}{c} - \varphi\right) &= \Gamma\left(\kappa\left(1 - \left(1-\beta\right)\left(1-d\right)bx^{\alpha}\frac{n}{c}\right) - 1\right) + \Gamma c^{-1}x^{\alpha}bn\tau\left(1-\beta\right)\left(1-d\right) \\
\Leftrightarrow\quad \Gamma^{-1}\frac{c}{g}\left(\frac{g}{c} - \varphi\right) &= (\kappa - 1) - (\kappa - \tau)\left(1-\beta\right)\left(1-d\right)c^{-1}x^{\alpha}bn \qquad (39)
\end{aligned}
$$

We further use (33) to rewrite κ as $\kappa = \frac{(1-(1+\vartheta)bx^{\alpha})+bx^{\alpha}(1-d)(1+(1-x)\vartheta)}{(1-(1+\vartheta)bx^{\alpha})+bx^{\alpha}(1-d)(\beta+(1-x)\vartheta)}$, and apply the latter to eliminate κ in (39)

$$\begin{aligned} & \Gamma^{-1}\frac{c}{g}\left(\frac{g}{c}-\varphi\right) \qquad (40) \\ = \; & \left(x^{\alpha}b\left(1-\beta\right)\left(1-d\right)\right)\frac{n}{c} \\ & \cdot \frac{\frac{c}{n}-\left[\left(1-\tau\right)\left(1-(1+\vartheta)bx^{\alpha}\right)+bx^{\alpha}(1-d)\left(\left(1+(1-x)\vartheta\right)-\tau\left(\beta+\left(1-x\right)\vartheta\right)\right)\right]}{\left(1-(1+\vartheta)bx^{\alpha}\right)+bx^{\alpha}(1-d)\left(\beta+\left(1-x\right)\vartheta\right)} \end{aligned}$$

We finally combine (30) and (31) to get $\frac{c}{n} = 1 - bx^{\alpha}d - \tau\left(1 - bx^{\alpha}\left(1-\beta(1-d)\right)\right)$, and replace c/n in (40) and well as $(1-(1+\vartheta)bx^{\alpha})$ with (33). Rearranging terms then yields

$$\frac{g}{c} = \varphi + q \cdot \Gamma\frac{g}{c^2}\frac{bx^{\alpha}\vartheta\left(1-\tau\right)\left(d\left(1-x\right)+x\right)}{\frac{c\chi}{1-\tau}+x^{\alpha}b\vartheta\left(1-x\right)\left(1-\beta\right)\left(1-d\right)} \qquad (41)$$

where we used $q = nx^{\alpha}b\left(1-\beta\right)\left(1-d\right)$.

From this, the case of no labor market frictions due to zero hiring costs, $b \to 0$, can be seen to imply

$$\frac{g}{c} = \varphi, \qquad (42)$$

as claimed in section 4.1.

Defining $\Phi = \Gamma\frac{g}{c^2}\frac{bx^{\alpha}\vartheta(1-\tau)(d(1-x)+x)}{\frac{c\chi}{1-\tau}+x^{\alpha}b\vartheta(1-x)(1-\beta)(1-d)}$, (41) can be written as $g/c = \varphi + q\Phi$ (see 23). To identify how labor market frictions affect the optimal tax rate, we substract both sides of $n\left(1-bx^{\alpha}d\right) = c+g$ (see 30) from $g/\tau = n\left(1-bx^{\alpha}\left[1-\beta(1-d)\right]\right)$ (see 31) to get $g\tau^{-1} - (g+c) = -nx^{\alpha}b\left(1-\beta\right)\left(1-d\right)$. Further using the definition of firm profits

$$q = nx^{\alpha}b\left(1-\beta\right)\left(1-d\right) \qquad (43)$$

and rearranging terms, shows that the tax rate under optimal policy satisfies

$$\tau = \frac{(g/c)}{1+(g/c)-qu_c}$$

which by eliminating g/c with (23) can be rewritten as (25). From this, and combining with (42) and (43), the case of no labor market frictions due to zero hiring costs, $b \to 0$, can be seen to imply

$$\tau = \frac{\varphi}{1+\varphi},$$

as claimed in section 4.1.

Finally, (33) can be used to show how labor market frictions affect the optimal consumption choice of the private sector:

$$\begin{aligned} c &= \chi^{-1}(1-\tau)(1-bx^{\alpha}(1-\beta(1-d)+\vartheta(1-\beta(1-d)(1-x)))) \\ &= \chi^{-1}(1-\tau)(1-z\cdot\Psi) \end{aligned}$$

where $\Psi(\beta, d, \vartheta, n) = 1-\beta(1-d)+\vartheta[1-\beta(1-d)\frac{1-n}{1-(1-d)n}] > 0$, which gives (26). In the case of no labor market frictions due to zero hiring costs, $b \to 0 \Rightarrow z = 0$, this simply yields $c = \chi^{-1}(1-\tau)$, as claimed in section 4.1.