Optimal Capital Structure in the Presence of Financial Assets*

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Abstract

Trade-off theories of capital structure describe how a firm chooses its leverage for a given set of assets. This paper studies how the predictions of such theories change if one accounts for the possibility that firms can invest in financial markets. Studying four different trade-off theories, the paper shows for each of them: given any set of firm assets and the corresponding optimal capital structure, the firm can reduce its leverage and its insolvency risk relative to this supposed optimum without a loss of value, if it 'integrates a fund' – i.e., if it issues additional equity in order to buy financial assets with certain properties. The paper thus indicates a way how the leverage and the insolvency risk of banks can be reduced without any costs in the long run.

JEL codes: G32, G30, G28, G10

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1 Introduction

Capital requirements are a key instrument for the regulation of banks, and their potential costs are a key issue in debates about financial regulation. Arguments for the existence of costs of capital requirements in the long run are based on theories that predict an optimal capital structure, which can be disturbed by such requirements.[1] These theories deviate from Modigliani and Miller (1958) by describing a trade-off between the respective costs of equity and debt financing. The classic example is the trade-off between taxes and bankruptcy costs (see Modigliani & Miller (1963) or Kraus & Litzenberger (1973)). In these trade-off models, the optimal capital structure always depends on the characteristics of the firm assets.[2] And in their description of the firm problem, these models always take the set of available assets as given. If one accounts for the fact, however, that firms can invest in financial markets, the set of available assets is not given on the firm level, but it depends on the decisions of other agents in the market. In particular, the set is not fixed, if new assets can be created by writing financial contracts. The aim of this paper is to show for several trade-off theories how their predictions about the optimal capital structure and the private costs of capital requirements change significantly, if one takes account of the possibility to invest in financial markets.

To put it differently, this paper analyze how the optimal capital structure of a firm changes, when it 'integrates a fund'. This means that I examine the capital structure of a firm which can choose to passively hold securities that are issued in the same financial market in which the firm issues its own debt and equity. I restrict the set of possible securities to simple financial assets whose payoffs only depend on the payoffs of firms in the market - like debt and equity claims or CDS. This implies that I do not consider a set of complete contracts that can condition on the processes within the firms and that could directly remove the frictions described by the trade-off theories. The frictions and the capital structure of the firm thus matter. But the possibility to invest in simple financial assets has important consequences.

I do not only consider the trade-off between taxes and bankruptcy costs, but also the one between different types of agency costs highlighted by Jensen and Meckling (1976). And I also address theories that are specific to banks and that try to explain their particularly high leverage - either by the disciplining role of demandable debt, as in Diamond and Rajan (2000), or by a premium of safe, 'money-like' claims, as in DeAngelo and Stulz (2015) or Gorton and Winton (2014).

For these four theories of capital structure I show: given any set of assets and the optimal capital structure that a firm chooses given these assets, the firm can reduce its leverage

[1] If capital requirements deviate from the optimal capital structure, banks incur private costs. And these can lead to social costs, if they impair the provision of credit and banking services to the economy. According to DeAngelo and Stulz (2015), capital requirements can cause social costs even directly by reducing the volume of 'money-like claims'. I will come back to this argument in more detail later.

[2] In case of a trade-off between taxes and bankruptcy costs, for instance, firms with less risky assets use more debt, because it reduces taxes while the expected costs of bankruptcy are small at the margin.

and its insolvency risk relative to this supposed optimum without a loss of value by means of an ‘integrated fund’. This means: by passively holding financial assets of the type described above which are issued in the same market like its own debt and equity. In fact, the integration of a fund that reduces insolvency risk can even lead to private gains for the firm. Let me briefly preview why (and under which conditions) this result holds for the different theories of capital structure, before I explain why firms might not use integrated funds in spite of their benefits.

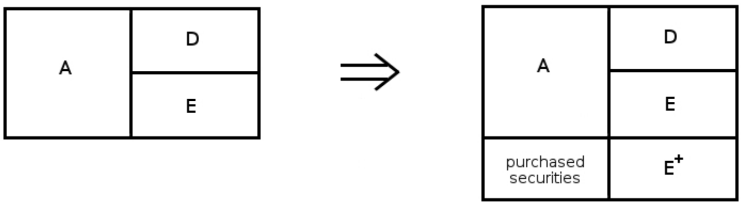

Figure 1: The 'integration of fund': a balance sheet with assets A, which are financed with debt D and equity E, is enlarged by purchasing financial assets. This purchase can be financed by, for instance, issuing new equity E^+.

Consider a firm that chooses its optimal capital structure for a given set of assets in the presence of a trade-off between taxes and bankruptcy costs. Assume now that this firm issues more equity in order to purchase financial assets in the same market, as illustrated in Fig. 1. The reduction of the firm leverage by means of such an 'integrated fund' can lead to private gains rather than losses, if the resulting reduction of the bankruptcy costs is larger than the increase in tax payments. This holds if the purchased financial assets have two properties: first, they have a sufficiently large value in states in which the firm without fund would become insolvent (so that they can avoid costly bankruptcies); second, their payoff in all other states is not too large (so that the increase in taxes is not larger than the decrease in bankruptcy costs). The possibility of a costless reduction of leverage and insolvency risk is thus due to the diversification that becomes possible with an integrated fund. Advantages of a diversification by means of financial assets have already been identified by the literature on hedging, see Smith and Stulz (1985), for instance.[3] In contrast to that literature, however, this paper shows that passively holding financial assets in an ‘integrated fund’ enables firms to decrease their leverage and insolvency risk without any of the costs that such changes of the capital structure supposedly entail. Let me briefly explain why this result also holds for other trade-off theories, before I discuss the availability of financial assets with the properties mentioned above.
According to Diamond & Rajan (2000) and their model of debt as a disciplining device, the optimal capital structure is the result of a trade-off that is very similar to the trade-off between taxes and bankruptcy. If the debt can be withdraw quickly, it can stop managers that try to extract rents from the firm payoff. Consequently, an increase of the debt

[3] A similar mechanism has also been identified for mergers, see e.g. Lewellen (1971).

level reduces this rent extraction (like it reduces taxes). A higher debt level, however, entails a higher probability that a low firm payoff triggers a run on the firm and leads to costly liquidations (similar to bankruptcy costs). Given the similar form of the trade-offs, the results are also similar: a reduction of leverage and insolvency risk by means of an integrated fund allows for gains rather than losses, if the payoff of the financial assets is such that the reduction in liquidation costs is larger than the increase in extracted rents. The effect of an integrated fund on the provision of safe, money-like claims is weakly positive, unconditionally. An integrated fund does not reduce the volume of debt issued by the firm.[4] But it weakly increases the safety of the debt owing to the additional payoff from the purchased assets. Thus, if there is a premium for issuing safe debt, the effect of integrated funds on the firm value is weakly positive.

In case of a trade-off between agency costs of debt (due to risk-shifting) and equity (due to reduced manager effort), as described by Jensen and Meckling (1976), the effect of an integrated fund depends on the payment scheme of the managers. As long as the payment does not condition on the payoff of the financial assets which are passively held in the fund, then the managers' incentives to exert effort or to engage in risk-shifting are not changed by the fund. Consequently, the integration of fund entails neither gains nor losses for the firm. But it stills reduces the leverage and the insolvency risk.

Summing up this preview, there is only one critical condition for the costless reduction of leverage and insolvency risk by means of integrated funds: the availability of financial assets with an appropriate distribution of payoffs. In order to study the availability of such assets, one could empirically investigate the properties of all outstanding financial assets. But this extensive investigation would only provide an incomplete answer, because it is always possible to create additional financial assets by writing new contracts. In fact, there is always a way to create financial assets with the properties that have been identified as sufficient conditions for the costless reduction of leverage and insolvency risk. I properly explain this creation in Section 6, but let me already indicate that it consists of a contract between the firm and an investment fund that is very similar to the 'liability holding company' (LHC) that has been proposed by Admati et al. (2012).

This paper and Admati et al. (2012) thus arrive at similar conclusions. But they derive these conclusions in different ways. Admati et al. (2012) provide qualitative arguments why LHCs allow for socially beneficial increases in capital requirements that do not disturb the corporate governance of banks, but rather improves it. This paper, in contrast, starts with a systematic analysis of optimal capital structures in the presence of financial assets. And studying four different trade-off theories, I show that integrated funds allow for private gains and that LHCs are a particularly beneficial type of integrated fund.

While integrated funds decrease the insolvency risk on the firm level, one might wonder how

[4] Gorton and Winton (2014) and DeAngelo and Stulz (2015) already indicate that equity-financed purchases of securities do not reduce the level of safe debt. But they are skeptical towards the ability of such purchases to reduce the bankruptcy risk. In Section 8, I show that comparably small funds already provide a significant increase of loss-absorbing capital, even in states with the worst aggregate risk.

the insolvency risk of firms changes in the aggregate. The cash flow from the financial assets held in integrated funds has to be provided by other agents in the market. And the overall level of cash flows in the economy does not change by rearranging them. Nevertheless, integrated funds can decrease the insolvency risk of firms in the aggregate. If a fund is added to an existing firm without changing its debt level, the insolvency risk of the firm never increases, but can only decrease. At the same time, the solvency of the providers of cash flows to integrated funds does not deteriorate because the cash flows are sold to integrated funds instead of other agents. To put it differently: Integrated funds allow to 'channel' cash flows from different sources through the balance sheets of firms, where they have beneficial effects, before the final recipients receive these cash flows.
The results of this paper lead to a puzzle: given that integrated funds allow for private gains, why do firms not use integrated funds? As pointed out by Admati et al. (2018), there is an asymmetric distribution of the gains and losses from changes in the capital structure, if the firm has outstanding debt. In case of a reduction of firm leverage and insolvency risk, the gains (e.g. reduced bankruptcy costs) accrue to the holders of the outstanding debt, while the firm owners incur costs (e.g. higher taxes).[5] This asymmetric distribution also applies in case of changes in the capital structure which lead to net gains. Since the owners only incur the losses, they have no incentive to implement such a change. This asymmetric distribution of gains and losses between debt and equity holders can explain the fact that firms do not use integrated funds. But it does not negate the result that a decrease of leverage and insolvency risk by means of integrated funds increases rather than decreases the value of the firm. And the asymmetric distribution of the net gains is only temporary, since the equity can participate in the gains once the outstanding debt has matured or has been rolled over with adjusted prices. This implies: integrated funds allow for an increase of capital requirements for banks and a decrease of their insolvency risk in a way that leads to benefits for all agents in the long run.

The **remainder of the paper** is organized as follows: Section 2 illustrates the basic idea in a simple example. Section 3 analyzes the trade-off between bankruptcy costs and taxes, and it also accounts for a premium for safe debt. Section 4 addresses the argument for a disciplining role of demandable debt, and Section 5 studies the trade-off between agency costs of debt and equity. Section 6 discusses the availability of financial assets with beneficial characteristics, before Section 7 indicates why firms do not use integrated funds despite their benefits. Section 8 concludes with stressing the implications of the analysis for the regulation of banks.

[5] As highlighted by Admati et al. (2018), this asymmetric distribution holds even in absence of frictions like e.g. taxes. A reduction of the insolvency risk always implies that the expected payoff to holders of outstanding debt increases. And they do not pay for this increase, but they gain at the expense of the equity holders.

2 An Illustrative Example

This section uses a simple example to illustrate two key results of this paper. First, it demonstrates why an integrated fund allows for a costless increase of equity above the level that is supposedly optimal for a single firm according to a trade-off theory of capital structure. Second, it demonstrates why integrated funds can decrease the probability of insolvencies in an economy, although the net amount of available payoffs as well as the debt levels of the firms remain the same.

Assume that there are two firms, A and B, with assets that have a stochastic payoff at $t = 1$. There are three equally probable states $\{I, II, III\}$ at $t = 1$ and the state-dependent payoffs y_A and y_B of the respective firm assets are:

	state I	state II	state III
asset payoff of firm A (y_A)	90	100	110
asset payoff of firm B (y_B)	105	90	105

The correlation between the asset payoffs of both firms is zero. Assume that each firm is initially owned by an agent who sells equity and debt claims to the firm to a continuum of investors at $t = 0$ and who tries to maximize the revenue from this sale. For simplicity, let us think of the investors as a continuum of risk-neutral agents who are willing to buy a claim at $t = 0$ at a price that equals its expected payoff at $t = 1$ (which is equivalent to a risk-free interest rate $r = 0$). Both types of claims entail losses. On the one hand, the payoff of equity claims at $t = 1$ is reduced by a relative loss τ (like a tax, for instance), so that equity holders only receive the payoff $1 - \tau$ per unit of residual firm payoff. On the other hand, if the firm has to default on the debt at $t = 1$, the asset payoff is reduced by a firm-specific loss b_x with $x \in \{A, B\}$ (like bankruptcy costs, for instance). Given that the initial firm owners want to maximize the revenue from selling equity and debt claims at $t = 0$, their problem consists of choosing the face values D_x of the firm debt that maximize the expected payoff of the sold claims. Formally, the initial owner of firm $x \in \{A, B\}$ solves the problem

$$\max_{D_x \in \mathbb{R}} \frac{1}{3} \sum_{i=I}^{III} \left(y_x^i - \tau \max\{0, y_x^i - D_x\} - b_x \cdot \mathbf{1}_{\{y_x^i < D_x\}} \right).$$

In order to focus on an interesting case, let us impose

Assumption 1 *: $10\tau < b_A < 20\tau$ and $30\tau < b_B$.*

Lemma 1

If Assumption 1 holds, the optimal debt levels of the firms are $D_A = 100$ and $D_B = 90$. This choice implies that firm A becomes insolvent in state I.

Given the optimal choice, the state-contingent payoffs of assets, debt and equity are:

***Firm* A**	state I	state II	state III
asset payoff	90	100	110
debt payoff	$90-b_A$	100	100
equity payoff	0	0	$(1-\tau)10$

***Firm* B**	state I	state II	state III
asset payoff	105	90	105
debt payoff	90	90	90
equity payoff	$(1-\tau)15$	0	$(1-\tau)15$

Proof: The first term in the objective function (i.e., y_x^i) is independent of D_x. The second term (i.e., $-\tau \max\{0, y_x^i - D_x\}$) is continuously increasing in D_x and reaches it maximum when D_x equals the largest possible realization of y_x^i. And the last term (i.e., $-b_x \cdot \mathbf{1}_{\{y_x^i < D_x\}}$) decreases with increasing D_x by discrete steps at each possible y_x^i. Consequently, the relative maxima of the objective function are at $D_x = y_x^i$ for $i \in \{I, II, III\}$. For firm A, the absolute maximum is at $D_x = 100$, since switching to $D_x = 90$ changes the objective function by $\frac{1}{3}\, b_A - \frac{1}{3}\, \tau\, 20 < 0$, and switching to $D_X = 110$ changes the objective function by $-\frac{1}{3}\, b_A + \frac{1}{3}\, \tau\, 10 < 0$. For firm B, the absolute maximum is at $D_x = 90$, since switching to $D_x = 105$ changes the objective function by $-\frac{1}{3}\, b_B + \frac{1}{3}\, \tau\, 30 < 0$.

Lemma 1 states the capital structures that are optimal for the firms, if one considers each firm separately. Let us now account for the possibility of an 'integrated fund'. Consider that firm A reduces its debt-to-equity ratio (relative to the case described in Lemma 1) by issuing more equity and investing the proceeds in securities issued by firm B. More precisely, consider the following. At $t = 0$, when firm B sells debt with face value $D_B = 90$ as well as its equity, firm A buys the fraction $\frac{10}{(1-\tau)15}$ of the equity of firm B. Given prices that equal the expected payoff of the claims, firm A has to pay $\frac{10}{(1-\tau)15} \cdot \big(\frac{1}{3}\,(1-\tau)15 + \frac{1}{3}\,(1-\tau)15\big) = \frac{20}{3}$ for this fraction of equity. The purchase implies that the portfolio of firm A is enlarged. The state-contingent payoff of the enlarged portfolio is the sum of the payoff of the firm assets plus the payoff of the fraction $\frac{10}{(1-\tau)15}$ of the equity of firm B (both are stated in the tables above). Given the enlargement of the portfolio, firm A can sell a more valuable set of claims to the investors. Let us assume, however, that firm A does not change its debt level relative to the benchmark case given in Lemma 1, which means that it sells debt with face value $D_A = 100$. Given this 'integration of a fund', the state-contingent payoffs of firm portfolio, debt and equity claims are:

***Firm* A** *(incl. fund)*	state I	state II	state III
portfolio payoff	100	100	120
debt payoff	100	100	100
equity payoff	0	0	$(1-\tau)20$

Observation 1
The integration of a fund (which means that firm A enlarges its portfolio by buying equity of firm B without increasing its debt level D_x) leads to the following changes relative to the benchmark case determined in Lemma 1 given Assumption 1:

1. *The fund is efficient as well as privately beneficial for firm A: the price/expected*

payoff $\frac{20}{3}$ *of the equity of firm B held by firm A is smaller than the increase in the expected payoff of claims that firm A sells to investors, which is* $\frac{1}{3}(10+b_A)+\frac{1}{3}(1-\tau)10$.

2. *The leverage of firm A decreases: while the debt level remains* $D_A = 100$, *the expected payoff of the firm portfolio increases from* 100 *to* $\frac{320}{3}$*; and in terms of expected payoffs of the claims, the debt-to-equity ratio decreases from* $\frac{290-b_A}{(1-\tau)10}$ *to* $\frac{300}{(1-\tau)20}$.

3. *The probability of firm insolvencies decreases: firm A does no longer become insolvent in state I, while the insolvency probability of firm B remains zero.*

The reduction of leverage and insolvency risk by means of an integrated fund differs from a reduction of these parameters by means of a decrease in the debt level. The latter only affects the liability side, while the former changes the asset side as well as the liability side. The integration of a fund has two positive effects on the solvency of the firm: it does not only reduce the firm leverage (as a debt reduction does), but it also allows for an improved diversification of the portfolio.[6] The fund can prevent an insolvency, if the payoff of purchased financial assets is sufficiently large in those states in which the payoff of the firm assets is too low to pay off the debt, as in case of firm A and state I. As a consequence, an integrated fund can be more efficient in decreasing the insolvency probability than a simple debt reduction: if the insolvency risk of firm A is reduced to zero by reducing the firm debt from the optimal level $D_A = 100$ to $D_A = 90$, then the equity value increases by $\frac{1}{3}(1-\tau)20$ and the corresponding increase $\frac{1}{3}\tau\, 20$ in taxes dominates the reduction $\frac{1}{3}b_A$ in bankruptcy costs; if the insolvency risk of firm A is reduced to zero by an integrated fund as described above, then the equity only increases by $\frac{1}{3}(1-\tau)10$ and the increase $\frac{1}{3}\tau\, 10$ in taxes is smaller than the reduction $\frac{1}{3}b_A$ in bankruptcy costs.

The result has some similarity to the argument of Lewellen (1971) that mergers can be beneficial owing to the diversification which they entail. But the example here shows that the benefits from diversification can already be obtained by just holding some securities issued by another firm instead of completely merging with that firm. Furthermore, the mechanism highlighted here does not depend on the fact that the purchased assets are equity claims of another firm, but it holds for any financial assets with an appropriate distribution of payoffs. The mechanism is related to mechanisms that have been discussed in the literature about hedging (see e.g. Smith and Stulz (1985)). The distinguishing feature of the example discussed here (and of this paper in general) is that it shows how the diversification benefits from the purchase of financial assets can be used to reduce the leverage and insolvency risk of firms without private or social costs.

The example demonstrates that the aggregate insolvency risk in the economy can decrease owing to an integrated fund, although neither the debt liabilities D_A and D_B nor the payoffs y_A and y_B of the underlying assets change. The integrated fund only redirects the cash flows from the assets before they are received by the investors. Some part of the

[6] This result does not rely on a strong negative correlation of the firms and their assets, as demonstrated by this example, in which the correlation between the asset payoffs is zero.

payoff of firm B is not directly paid to the investors, but the investors receive it via the balance sheet of firm A. Since the debt level of firm B does not change, this redirection does not change its insolvency risk. But the redirection prevents the insolvency of firm A in state I, and it thus also prevents the related costs. To sum up, integrated funds allow to 'channel' cash flows from firms through the balance sheets of other firms, where they have beneficial effects, before they arrive at the final recipients.
The fact that integrated funds allow for a costless decrease of leverage and insolvency risk is not a particular feature of the example studied here. The next sections show that this result holds for more general cases of firms and for different trade-off theories. The result that integrated funds do not only reduce the insolvency risk at the firm level but also in aggregate is generalized in Appendix B.

3 Taxes, Bankruptcy Costs, and Safe Debt

This section shows that the possibility to costlessly reduce the insolvency risk of a firm by means of an integrated funds holds for any firm that faces a trade-off between taxes and bankruptcy costs (as described in Modigliani & Miller (1963) or Kraus & Litzenberger (1973), for instance). In addition, I account for a premium for safe debt, as suggested by DeAngelo & Stulz (2013) and Gorton & Winton (2014). This premium is very similar to the tax benefit of debt, apart from its restriction to a certain subset of the firm debt.[7] This section does not provide a complete solution of the firm problem (which depends on the financial assets offered by other firms), but it indicates that each firm can gain from an integrated fund, given that financial assets with certain features are available on the market. The availability of such assets and the potential puzzle that firms might not use integrated funds despite the gains is discussed in the Sections 6 and 7.
Consider an owner of a firm with assets that yield a stochastic payoff $R \in \mathbb{R}^+$ at $t = 1$. Besides these firm-specific assets, the firm can also 'integrate a fund', which means that it can buy a set S of financial assets in the same financial market in which it issues its own debt and equity. I will comment on the choice of S later, but let us first assume that the composition of S is given and that the firm only chooses the amount s it invests in this portfolio at $t = 0$. The portfolio yields a stochastic cash flow $R_S \in \mathbb{R}^+$ at $t = 1$ per unit of s. The joint distribution of R and R_S is continuous and denoted as $\hat{f}$. The univariate marginal distribution of R is $f(R) := \int \hat{f}(R, R_S)\, dR_S$.
At $t = 0$, the initial firm owner issues equity and two types of debt claims: senior debt with safe payoff D_s at $t = 1$, and junior debt with face value D_r and default probability ϕ. Assume that the firm has no outstanding debt at $t = 0$. The probability that the firm

[7] According to Gorton and Pennacchi (1990), safe debt is useful as a means of payment and investors thus accept a discount on the interest rate of such claims.

become insolvent at $t = 1$ is

$$\phi(D_s, D_r, s) = \int \mathbf{1}_{\{R + s\, R_S < D_s + D_r\}}\, \hat{f}(R, R_S)\, dR_S\, dR\,.$$

Let us define the leverage $l(D_s, D_r, s)$ of the firm as the ratio of the face value of its debt over the expected cash flow of its assets: $l(D_s, D_r, s) = \frac{D_s + D_r}{E_{\hat{f}}[R + s\, R_S]}$

Observation 2

If the debt level $D_s + D_r$ is fixed, an increase in the size s of the integrated fund leads to:

- *a decrease of the probability of insolvency: $\frac{d}{ds}\phi(D_s, D_r, s) \leq 0\ \forall\ s \in \mathbb{R}^+$, with a strict inequality for some $s \in \mathbb{R}^+$ if $E_{\hat{f}}\left[\mathbf{1}_{\{R_S > 0\}}\mathbf{1}_{\{R < D_s + D_r\}}\right] > 0$;*
- *a decrease of the leverage: $\frac{d}{ds} l(D_s, D_r, s) < 0\ \forall\ s \in \mathbb{R}^+$.*

Assume that the objective of the initial owner is to maximize the revenue from selling the equity and debt claims at $t = 0$. For simplicity, assume that the claims are priced in competitive markets with risk-neutral investors and a risk-free interest rate $r = 0$. (Appendix C shows the robustness of the results to more general preferences of investors.) Assume that all agents can observe the firm's choice of capital structure and know $\hat{f}$ at $t = 0$. With b denoting bankruptcy costs that reduce the asset payoff in the event of insolvency, the value d_r of the junior debt sold at $t = 0$ is given as

$$d_r(D_s, D_r, s) = \big(1 - \phi(D_s, D_r, s)\big) D_r + \int \mathbf{1}_{\{R + s\, R_S < D_s + D_r\}} \big(R + s\, R_S - D_s - b\big)\, \hat{f}(R, R_S) dR_S dR$$

To account for the premium of safe debt, let us assume that the claim is priced with a reduced interest rate $r_s = -\frac{\lambda}{1+\lambda}$. A microfoundation of the premium for safe debt is given in Appendix **??**. The value d_s of the safe debt at $t = 0$, which is the discounted value of the safe payoff D_s, is then

$$d_s(D_s) = \frac{1}{1 + r_a} D_s = \frac{1}{1 - \frac{\lambda}{1+\lambda}} D_s = (1 + \lambda)\, D_s\,.$$

To account for the tax benefit of debt, let us assume that the tax payments of the firm are given by $T(y_e)$ with $T'(y_e) > 0$ and y_e as residual payoff $y_e = \max\{0, R + s\, R_S - D_r - D_s\}$. The value e of the equity at $t = 0$ is the expected residual payoff net of taxes:

$$e(D_s, D_r, s) = \int \max\{R + s\, R_S - D_r - D_s, 0\}\, \hat{f}(R, R_S)\, dR\, dR_S - T^{exp}(D_r, D_S, s)\,,$$

with $T^{exp}(D_r, D_S, s) := \int T\big(\max\{R + s\,R_S - D_r - D_s, 0\}\big)\,\hat{f}(R, R_S)\,dR\,dR_S$. The 'firm value' v_s, which means the joint value of the equity and debt claims at $t = 0$, is

$$
\begin{aligned}
v_s(D_r, D_s, s) &= d_r(D_r, D_s, s) + d_s(D_s) + e(D_r, D_s, s) \qquad (1)\\
&= \int \big(R + s\,R_S\big)\,\hat{f}(R, R_S)\,dR\,dR_S - T^{exp}(D_r, D_S, s) - b\,\phi(D_s, D_r, s) + \lambda\,D_s
\end{aligned}
$$

The 'net firm value' v, which means the firm value v_s net of the expected payoff $E_{\hat{f}}\,[s\,R_S]$ of the financial assets held in the integrated fund, is:

$$
\begin{aligned}
v(D_r, D_s, s) &= v_s(D_r, D_s, s) - E_{\hat{f}}\,[s\,R_S] \qquad (2)\\
&= \int R\,\hat{f}(R, R_S)\,dR\,dR_S - T^{exp}(D_r, D_s, s) - b\,\phi(D_s, D_r, s) + \lambda\,D_s
\end{aligned}
$$

Assumption 2 *(no-arbitrage-condition)*
a) The price of the financial assets at $t = 0$ *equals their expected payoff* $E_{\hat{f}}\,[s\,R_S]$ *at* $t = 1$.
b) The outcome $R_S = 0$ *has strictly positive measure.*

Assumption a) is imposed in order to study the case that the firm purchases financial assets in the same market in which it issues its own claims, where the riskfree rate is $r = 0$. Assumption b) excludes the purchase of financial assets with a safe payoff. It is only imposed to simplify further notation. If $R_S > 0$ in all states, the safe part of this payoff would be priced in terms of the reduced rate r_a, but this premium would net out with the premium of the claims that the firm issues against this portfolio.
If the initial owner wants to maximize the revenue from the sale of claims at $t = 0$ net of the costs of purchasing the financial assets, then her decision problem is:

$$
\max_{s\in\mathbb{R}^+,\,D_s\in[0,\overline{D}_s],\,D_r\in\mathbb{R}^+} \Big(v^s(D_s, D_r, s) - E_{\hat{f}}\,[s\,R_S]\Big) = \max_{s\in\mathbb{R}^+,\,D_s\in[0,\overline{D}_s],\,D_r\in\mathbb{R}^+} v(D_s, D_r, s)\,,
$$

with $\overline{D}_s := \min\big(R + sR_S \,|\, \hat{f}(R, R_S) > 0\big)$ as the lowest possible firm payoff. The problem of the firm owner thus consists of maximizing the net firm value v. In order to discuss how the integration of a fund affects the net firm value, let us define the constrained problems

$$
\begin{aligned}
&\textit{Problem } P(s): \max_{D_s\in[0,\overline{D}_s],\,D_r\in\mathbb{R}^+} v(D_s, D_r, s) \text{ for given } s \in \mathbb{R}^+,\\
&\textit{Problem } P(s, D): \max_{D_s\in[0,\overline{D}_s],\,D_r\in\mathbb{R}^+} v(D_s, D_r, s) \text{ s.t. } D_s + D_r = D, \text{ for given } s \in \mathbb{R}^+.
\end{aligned}
$$

The solution of $P(0)$ is the capital structure that the firm owner chooses, if there is no possibility of an integrated fund. It shall be denoted as $(D_{s,0}, D_{r,0})$, and $D_0 := D_{s,0} + D_{r,0}$. The solution of $P(s, D)$ is the combination of safe and risky debt that the firm optimally sells, if the joint face value $D_s + D_r$ of the debt is fixed at D and the firm has an integrated fund with size s. This solution shall be denoted as $(D_s(s, D), D_r(s, D))$.

Proposition 1

a) Relative to the optimal capital structure of a firm without integrated fund, a reduction of the leverage and insolvency risk by means of an integrated fund increases the net firm value, if the payoff of the purchased financial assets is such that Eq. (3) *holds:*

$\frac{d}{ds} v(D_{s,0}, D_{r,0}, s)\big|_{s=0} > 0$*, if*

$$b\, E_{\hat{f}}\left[R_S | R = D_0\right]\, f(D_0) \;>\; \int R_S\, T'(R - D_0)\, \hat{f}(R, R_S)\, dR\, dR_S\,. \tag{3}$$

b) An integrated fund weakly increases the minimal possible payoff of the firm and thus weakly increases the fraction of the firm debt D_0 that can be sold as safe debt. Accounting for this, an integrated fund already increases the net firm value, if Eq. 4 holds:

$$\frac{d}{ds} v(D_s(s, D_0), D_r(s, D_0), s)\bigg|_{s=0} > 0, \quad \textit{if} \tag{4}$$

$$b\, E_{\hat{f}}\left[R_S | R{=}D_0\right] f(D_0) + \lambda \,\min\left(R_S | \hat{f}(\underline{R}, R_S){>}0\right) > \int R_S\, T'(R{-}D_0)\, \hat{f}(R, R_S)\, dR\, dR_S\,,$$

with $\underline{R}$ representing the lower bound $\min(R|f(R) > 0)$ *for R.*

Proof: See Appendix D.1.

The explanation for this result is the same as in the simple example presented in the previous section. Given an optimal capital structure $(D_{s,0}, D_{r,0})$ of the firm without fund, a reduction of insolvency risk and leverage by means of a debt reduction leads to a decrease in the firm value. A reduction of leverage and insolvency risk by means of an integrated fund, in contrast, can increase the firm value, because it can be more efficient in decreasing the insolvency probability than a simple debt reduction. The integration of a fund has two positive effects on the solvency of the firm: besides reducing the firm leverage (which a reduction of the debt level could also achieve), it allows for an improved diversification of the firm portfolio. If the payoff of purchased financial assets are sufficiently large in those states in which the firm-specific assets yield relatively low payoffs, the bankruptcy of the firm can be prevented. If the resulting reduction in expected bankruptcy costs (given by the l.h.s. of Eq. 3) is larger than the increase in tax payments due to an increased payoff to the firm equity (given by the r.h.s. of Eq. 4), then the firm value increases.

If there is a premium for safe debt, there is an additional positive effect of the integrated fund. If the payoff of the purchased assets is greater than zero in all states in which the payoff of the initial firm assets equals the minimal possible value $\underline{R}$ (which means if $\min\left(R_S | \hat{f}(\underline{R}, R_S) > 0\right) > 0$), the minimal payoff of the firm portfolio increases. (This is possible in spite of Assumption 2 b, since the worst realizations of both sets of assets, $R_S = 0$ and $R = \underline{R}$, do not necessarily occur in a same state.) If the minimal payoff increases, the firm with integrated fund can choose a higher level of safe debt, which implies a larger premium[8].

Proposition 1 holds for any set of firm assets and corresponding optimal capital structure. For each possible distribution f of the payoff R, Eq. (3) specifies a sufficient condition for a costless reduction of leverage and insolvency risk by means of an integrated fund. More precisely, the reduction of leverage and insolvency risk is not only costless, but it even increases the firm value. The condition refers to properties of the joint distribution $\hat{f}(R, R_S)$. In principle, one can always construct a financial asset with an appropriate distribution:

Lemma 2

For every firm with continuous distribution $f(R)$ of its asset payoff and a corresponding optimal capital structure with strictly positive bankruptcy risk (i.e. $\phi(D_{s,0}, D_{r,0}, 0) > 0$), there is a financial asset whose payoff R_S is distributed such that Eq. (3) holds.

Proof by example: Consider a financial asset that yields a cash flow $R_S = \frac{1}{m}$ in all states with $R \in [0, D_0]$ and zero in all other states, with m being a normalization factor such that $E_{\hat{f}}[R_S] = 1$. For this asset, the l.h.s. of Eq. 3 is strictly positive, and the r.h.s. is 0.

The possibility to construct an appropriate financial asset is a simple, theoretical result. The more interesting and relevant question is whether one should expect that financial assets with appropriate characteristics are actually offered by other agents in the market. I discuss this question in Section 6. And Appendix B shows that the results obtained in this section are robust on aggregate level. This means that all firms in an economy can simultaneously benefit from integrated funds and can reduce their insolvency risk, although the underlying real assets of the economy remain the same.

4 Disciplining Role of Demandable Debt

This section shows that the possibility to costlessly reduce leverage and insolvency risk of a firm by means of an integrated fund is not a particular feature of the trade-off between bankruptcy costs and debt benefits, but that it holds for other trade-off theories as well. This shall be illustrated for a theory that has been used to justify the high leverage of the banking sector. Calomiris and Kahn (1991) and Diamond and Rajan (2000) have argued that a fragile funding structure with high levels of demandable debt can be optimal, because it disciplines the managers by the threat of 'runs' and reduces their possibilities to extract rents from the cash flow to investors. As the last section, this section does not provide a complete solution of the firm problem, but it indicates that each firm can gain from an integrated fund, given that financial assets with certain features are available on the market. The availability of such assets and the potential puzzle that firms might not

[8] This effect has already been indicated in Admati et al. (2013). Gorton and Winton (2014) neglect this effect in their analysis of the premium for safe debt, because they assume perfect correlation between all issuers of financial claims. I illustrate in Section 8 that their strict assumption is an inappropriate simplification, even if one considers the portfolio of banks in the worst crises.

use integrated funds despite the gains is discussed in the Sections 6 and 7.

Let us keep the same basic structure of the firm problem as in the previous section. There is a firm with a set of firm-specific assets that yield $R \in \mathbb{R}^+$ at $t = 1$, and this firm can 'integrate a fund' in addition. This means that it can invest an amount s at $t = 0$ in a set S of financial assets which yield $R_S \in \mathbb{R}^+$ per unit of s at $t = 1$. The continuous joint distribution of R and R_S is denoted as $\hat{f}$. Let us assume that the financial assets are purchased in same market with risk-neutral pricing and $r = 0$ in which the firm issues its own debt and equity. Consequently, **Assumption 2 a)** still applies and the price of the financial assets at $t = 0$ equals $E_{\hat{f}}\,[s\,R_S]$. (Appendix C shows the robustness of the results to more general pricing kernels and preferences of investors.) The initial owner of the firm sells equity and debt claims $t = 0$ and chooses the capital structure such that it maximizes the revenue from these sales. The face value of debt is denoted as D, there is no outstanding debt at $t = 0$, and all agents know the firm's choice of capital structure as well as $\hat{f}$. The probability of insolvency at $t = 1$ is $\phi(D, s) = \int \mathbf{1}_{\{R+s\,R_S<D\}}\hat{f}(R, R_S)dRdR_S$, and the leverage is defined as $l(D, s) = \frac{D}{E_{\hat{f}}[R+s\,R_S]}$. The analogue of **Observation 2** also holds here: an increase in the size s of the integrated fund leads to a decrease of both, the leverage $l(D, s)$ and the insolvency probability $\phi(D, s)$.

For the purpose of this paper, it is sufficient to briefly summarize the story presented in Diamond and Rajan (2000) and to focus on the resulting trade-off in the choice of capital structure.[9] Assume that the firm is operated between $t = 0$ and $t = 1$ by managers who obtain special knowledge about the firm production. (In case of a bank, for instance, they establish lending relationships.) If the operation is not completed by the managers, but debt or equity holders take over at $t = 1$ and 'liquidate' the firm, the payoff of the firm-specific assets declines from R to $R - l\,R$ with $0 < l < 1$. It seems implausible that managers have a similar advantage in passively holding financial assets within the fund. For completeness, however, I consider the possibility that R_S declines to $(1 - l_S)R_S$ with $0 \leq l_S < 1$ in case of a liquidation.

The managers are able to extract a fraction of the firm payoff at $t = 1$, because the equity holders are better off with accepting such an extraction than with firing the managers and incurring the relative loss l. This rent extraction, however, can be constrained by debt in the form of depositors. The key characteristic of deposits is: when they are withdrawn at $t = 1$, they are paid out at face value in the order in which the withdrawal request arrive. The depositors therefore immediately run when the expected payoff of their claims is smaller than the face value D, either because $R + sR_S < D$ or because the managers attempt to extract some of their payoff. Since the action of the depositors is immediate

[9]The model focuses on the disciplining of the management by means of a fragile capital structure. It does not address the alleged potential of fragile funding structures to extract higher interest rates from the borrowers of banks. If one wanted to analyze comprehensively how the capital structure affects the extraction of cash flows from borrowers, one would need to go beyond Diamond and Rajan (2000), anyway. One would need to take into account, for instance, the reaction of borrowers to an increased extraction of rents that the fragile funding allows for (e.g. less entrepreneurial activity or evasion to alternative funding).

and uncoordinated, there is no chance for the managers to accomplish the extraction or to negotiate any other rent. The costs of this 'disciplining device' is the possibility of inefficient liquidations. The optimal capital structure trades off the relative losses $lR + l_S\, sR_S$ from the 'runs' of depositors against the extraction of rents by managers. In order to study this trade-off, one has to consider three types of states:

1. If $R + sR_S < D$, the depositors run on the firm and take hold of all assets. They only receive $R_l := (1-l)R + (1-l_S)s\, R_S$ due to an inefficient liquidation. Managers and equity holders get nothing.

2. If $R_l < D \leq R + sR_S$, the depositors can be sure that they receive D.[10] The managers do not dare to extract some of the payoff to depositors, because they would lose access to the remaining cash flow $R + sR_S - D$. The equity holders do not take over the firm, because they could only obtain the cash flow R_l and would hence face a run. The distribution of $R+sR_S-D$ between managers and equity holders depends on the bargaining game between them. Let $\tau_m \in (0, 1)$ simply represent the fraction that the managers obtain.

3. If $D \leq R_l$, the situation is similar to case 2. The depositors can be sure to get D and the equity holders and the managers bargain over the relative surplus that arises from keeping the managers. Since the equity holders could take over the firm without facing a run, the relative surplus is $lR+l_S\, sR_S$. Assume again that the managers get a fraction τ_m.

To sum up, the state-contingent payoffs at $t = 1$ are:[11]

Payoffs to	depositors	equity holders	managers
1. $R + sR_S < D$	R_l	0	0
2. $R_l < D \leq R + sR_S$	D	$(1-\tau_m)\big(R+sR_S-D\big)$	$\tau_m \cdot \big(R+sR_S-D\big)$
3. $D \leq R_l$	D	$R+sR_S-D-\tau_m \cdot (l\,R+l_S\, sR_S)$	$\tau_m \cdot (l\,R + l_S\, sR_S)$

The value v_s of the firm at $t = 0$ is defined as the joint value of the debt and equity claims. Given that the value of the claims at $t = 0$ equals their expected payoff at $t = 1$, the firm value $v_s(D, s)$ can be written as

$$
\begin{aligned}
v_s(D, s) &= \int (R + s\, R_S)\, \hat{f}(R, R_S)\, dR_S\, dR - L(D, s), \quad \text{with} \\
L(D, s) &= \int \tau_m \cdot \big(l\, R + l_S\, sR_S\big)\, \mathbf{1}_{\{D \leq R_l\}}\, \hat{f}(R, R_S)\ dR_S\, dR \\
&\quad + \int \tau_m \cdot \big(R + sR_S - D\big)\, \mathbf{1}_{\{R_l \leq D \leq R+sR_S\}}\, \hat{f}(R, R_S)\ dR_S dR \\
&\quad + \int \big(l\, R + l_S\, sR_S\big)\, \mathbf{1}_{\{R+sR_S \leq D\}}\, \hat{f}(R, R_S)\ dR_S\, dR
\end{aligned}
$$

[10] The argument for a beneficial role of demandable debt by Diamond and Rajan (2000) treats the demandable debt favorably, as it neglects the possibility of non-fundamental runs. Since I want to critically discuss their argument, I follow them and neglect this type of run.

[11] The payoff stated for depositors is the payoff of the entire group, while the individual payoffs vary in case of $R + sR_S < D$ due to the sequential order in processing the withdrawals.

The 'net firm value' $v(D,s)$, which is v_s net of the price of the purchased assets, is:

$$v(D,s) = v_s(D,s) - E_{\hat{f}}\left[s\,R_S\right] = \int R\,\hat{f}(R,R_S)\,dR_S\,dR - L(D,s)$$

If the initial owner wants to maximize the revenue from the sale of claims at $t=0$ net of the costs of purchasing the financial assets, then her decision problem is:

$$\max_{D\in\mathbb{R}^+,s\in\mathbb{R}^+}\left(v^s(D,s) - E_{\hat{f}}\left[s\,R_S\right]\right) = \max_{D\in\mathbb{R}^+,s\in\mathbb{R}^+} v(D,s) \;\Leftrightarrow\; \min_{D\in\mathbb{R}^+,s\in\mathbb{R}^+} L(D,s)\,.$$

The decision problem consists of the maximization of the net firm value v, which is equivalent to the minimization of the expected losses L. The optimal capital structure balances out the expected extraction of payoff by managers (given by the first and second term in $L(D,s)$) and the expected loss from runs of depositors (given by the third term in $L(D,s)$). In order to study the impact of an integrated fund, let us again define the constrained problem $P(s)$: $\max_{D\in\mathbb{R}^+} v(D,s)$ for given $s\in\mathbb{R}^+$. The solution of $P(0)$ shall be denoted as D_0 and it represents the optimal capital structure of the firm without integrated fund.

Proposition 2

Relative to the optimal capital structure of a firm without integrated fund, a reduction of the leverage and insolvency risk by means of an integrated fund increases the net firm value, if the payoff of the purchased financial assets is such that Eq. 5 holds:

$\frac{d}{ds}v(D_0,s)\big|_{s=0} > 0$, *if*

$$l\,D_0\,f(D_0)\,E_{\hat{f}}\left[R_S|R{=}D_0\right] > \tau_m\cdot\left(\int_{D_0}^{\frac{D_0}{1-l}} E_{\hat{f}}\left[R_S|R\right]\cdot f(R)\,dR + l_S\int_{\frac{D_0}{1-l}}^{\infty} E_{\hat{f}}\left[R_S|R\right] f(R)\,dR\right) + l_S\int_0^{D_0} E_{\hat{f}}\left[R_S|R\right] f(R)\,dR. \qquad (5)$$

Proof: See Appendix D.2.

The explanation for this result is similar to the one for the results in the two previous sections. Given the optimal debt level D_0 of the firm without fund, a reduction of insolvency risk and leverage by means of a debt reduction leads to a decrease in the firm value. A reduction of leverage and insolvency risk by means of an integrated fund, in contrast, can increase the firm value, because the fund can alter the distribution of the firm payoff in a beneficial way. In particular, the fund can prevent costly liquidations due to runs in states with $R < D$ by providing a sufficiently large payoff $s\,R_S \geq D - R$ in these states. The integrated fund increases the firm value, if the reduction in expected liquidation costs is larger than the rents that managers can extract from the fund payoff plus the relative loss from liquidating the fund in the remaining states with runs. This condition is expressed by Eq. (5) for a marginal increase of the fund size s: the l.h.s. states the reduction in costs from runs and inefficient liquidation, the first and second term on the r.h.s. state the extraction of fund payoffs by the managers, and the last term on the r.h.s. states the

expected loss from liqudating the fund in case of a run.
For any set of firm assets and corresponding optimal capital structure, Proposition 2 provides a sufficient condition for the possibility to decrease the leverage and insolvency risk of a firm without decreasing the firm value, but rather increasing it. In principle, one can always construct a financial asset with properties such that this condition is fulfilled:

Lemma 3
For every firm with continuous distribution $f(R)$ of its asset payoff and a corresponding optimal capital structure with strictly positive insolvency risk (i.e. with $\phi(D_0, 0) > 0$), there is a financial asset whose payoff R_S is distributed such that Eq. (5) *holds.*

Proof by example: Consider a financial asset with a state-contingent payoff $R_S = \frac{1}{m}\mathbf{1}_{\{R \leq D_0\}} + \frac{l_S}{l\, D_0\, f(D_0)}\mathbf{1}_{\{R=D_0\}}$, with m being a normalization factor such that $E_{\hat{f}}\left[R_S\right] = 1$. For this asset, the l.h.s. of Eq. 5 is $l\, D_0\, f(D_0)\frac{1}{m} + l_S$ and it is thus larger than the r.h.s. of Eq. 5, which is equal to $\tau_m \cdot 0 + l_S \cdot E_{\hat{f}}\left[R_S\right] = l_S$.

The possibility to construct an appropriate financial assets is a simple, theoretical result, and the more interesting question is whether other agents in the market can offer financial assets with such characteristics. This question is addressed in Section 6. This section has shown, however, that the possibility to costlessly reduce leverage and insolvency risk by means of an integrated fund is not a special feature of a single trade-off theory. It rather holds for different types of such theories, including the trade-off between rent extraction by managers and inefficient liquidations, which has been used to explain the high leverage of banks. Appendix B shows that this result is robust on the aggregate level, which means that it still holds when all firms simultaneously integrate funds.

5 Risk-Shifting and Effort Reduction

Having already discussed three theories of capital structure (disciplining role of debt, taxes vs. bankruptcy costs, premium for money-like claims), this section addresses another prominent theory of capital structure: Jensen and Meckling (1976) have argued that the optimal capital structure is determined by a trade-off between the respective agency costs of equity and debt financing. I only briefly indicate here why a firm can integrate a fund that reduces leverage and insolvency risk without disturbing the optimal trade-off between these agency costs. A more detailed and formal analysis of this issue is given in Appendix A.
According to Jensen and Meckling (1976), the capital structure of a firm affects the behavior of the firm managers in two ways, which have an impact on the firm value. First, the managers are paid by the firm with equity claims in order to incentivize them to exert costly effort[12] which increases the firm payoff and hence the payoff of the equity claims.

[12] Alternatively, 'effort' can be interpreted as the discipline to abstain from a privately beneficial misuse of firm resources.

This implies that the return to the manager effort is shared among all holders of equity. As a consequence, the incentive of the managers to exert costly effort decreases with an increasing amount of equity in excess of the claims that they receive as payment. This is the agency cost of equity. Second, being equity holders, the managers have an incentive to increase the risk of the firm portfolio after firm debt has issued. They gain from the upside risk, while the downside risk is partly borne by debt holders. If the increase of risk leads to a decrease in the mean firm payoff, this risk-shifting is inefficient. The incentive of the managers to engage in such risk-shifting increases with the debt level of the firm, because the part of the downside risk that the debt holders bear increases with the debt level. This is the agency cost of debt.

Assume that a firm with a set A of assets has chosen the debt level D, because this capital structure maximizes the firm value. This means that the effort c_m and the amount α of risk-shifting that the managers chooses in case of D leads to a higher firm value than the effort and risk-shifting that they would choose for any other level of debt. Consider now that this firms integrates a fund, which means: first, the firm buys a set S of financial assets in the same market in which it issues its own debt and equity; and second, in order to finance this purchase, the firm issues more of its own claims, but it does not change the face value D of the debt. The impact of the integrated fund on the firm value depends on its impact on the manager behavior (i.e., their choice of c_m and α). This impact depends on how the payment of the managers is adjusted to the integration of a fund. While the firm has many degrees of freedom in adjusting the payment scheme, I only present a simple example here, for which the integrated fund has neither a positive nor a negative effect on the firm value.

The key idea is to relate the payment of the managers to the part of the firm that actually depends on their behavior. This is the asset set A whose payoff R_A depends on the managers' effort c_m and their risk-shifting α. The payoff R_S of financial assets issued by other agents in the market is independent of the managers of the firm that only holds these assets passively. Assume that the managers are paid with a fraction m of an equity claim to the set A of assets which yields $\max\{0, R_A - D\}$. If the fraction m equals the fraction of equity that the managers would receive from the firm without fund, then the managers' return to effort is the same in both cases. Consequently, they choose the c_m and α that have been optimal for a firm without fund, given that the debt level D and the potential gains from risk-shifting have not changed. As a result, the payoff from the asset set A is not changed by the integrated fund. And the purchase of S financed with claims that are issued in the same market has zero net present value. This means that the net firm value does not change due to the integration of a fund. At the same time, however, the leverage of the firm (i.e., the ratio of its liability to the value of its portfolio) has decreases. And the same holds for its insolvency risk, as the payoff of the financial assets can be used to payoff D in states with low R_A.

The payment scheme just described relies on the possibility to separate between the payoffs of the asset sets S and A and the possibility to condition the manager payment only on the payoff of A. It should be very easy for the firm, however, to distinguish between the payoff R_S of the financial assets held in the fund and the payoff of the actual firm operation.

The adjustment of the payment scheme is an example of a more general point: although there are important relations between the capital structure of a firm and the incentives of agents in that firm, these relations are not as strict as first highlighted in Jensen & Meckling (1976). A firm has many degrees of freedom to shape these relations by writing better contracts with the involved agents. This has already been stressed for other types of problems by, for instance, Aghion & Bolton (1989) or Dybvig & Zender (1991).

6 A Way to Create Financial Assets with a Beneficial Distribution of Payoffs

The previous sections have identified sufficient conditions for the existence of efficiency gains owing to the integration of funds. These conditions are stated in Eq. (4) and Eq. (5) and refer to the joint distribution of the payoffs from the firm assets and from the purchased financial assets. The joint distribution has to be such that the financial assets yield relatively high payoffs in states in which the firm without fund would become insolvent. The key question is whether other agents in the financial markets provide securities that satisfy this condition.

One could empirically test for given firms whether there are outstanding assets in the financial markets that have an appropriate payoff distribution relative to the payoff distribution of those firms. If one finds financial assets with beneficial distribution of payoffs, one faces the puzzle why the firms do not 'integrate a fund' that holds these assets. Given the huge number of outstanding financial assets, however, this empirical exercise would be a vast task. And it would still provide an incomplete answer to the question, because the agents in the market can provide much more financial assets than the outstanding ones, because they can create new ones by simply writing contracts. I therefore address the question in a different way: I show that there is always a simple way to create financial assets with beneficial payoff distributions. This result leads to a puzzle, since this way of creating beneficial assets does not seem to be common practice. The puzzle is resolved in Section 7.

Consider that a firm with state-contingent payoff R of its assets and optimal debt level D purchases a 'capital insurance' that yields the payoff $\max\{D - R, 0\}$. This financial asset can be created by a very simple contract that simply condition on the payoff of the firm assets and on the face value of the debt liabilities. The capital insurance reduces the insolvency risk to zero, which also implies that is reduces the costs of bankruptcy or liquidations to zero. And it does not increase the payoff to equity, which means that it does not increases the taxes or the rents that managers can extract from the equity payoff.

(New agency problems that might arise from a capital insurance are discussed below and in the next section.) Consequently, the capital insurance increases the firm value according to the trade-off theories discussed in the previous sections.

From a practical point of view, it might be difficult to predict the payoff R in each possible state and it might be unfeasible to specify the payoff of the financial asset in each possible state. In that case, the insurance contract has to condition on the lack of payoff itself, which means the difference $D - R$. Such an insurance contract, however, leads to moral hazard, if the payoff R can be altered by the firm. Firm owners, for instance, have an incentive to engage in risk-shifting at the expense of the insurance provider. This means that they increase the volatility of their portfolio, so that they benefit from the increase upside risk, while the increased downside risk is covered by the insurance. In the following, I explain how the capital insurance can be provided in a way that prevents this type of moral hazard. The agency problem between the insurance provider and rent extracting managers (like in Section 4) is discussed in Section 6.1.

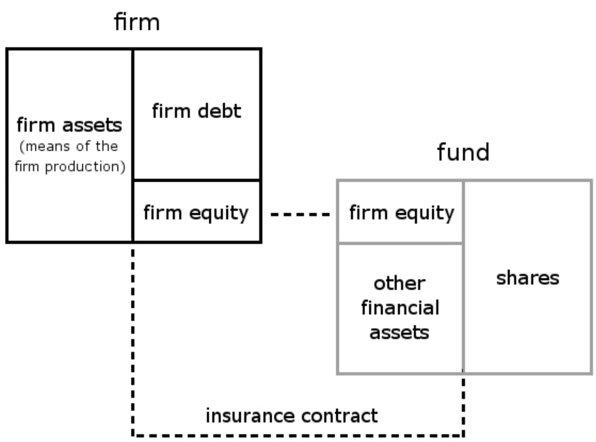

Figure 2: A possibility to provide a capital insurance to a firm without allowing for risk-shifting.

The way how a capital insurance can be provided to a firm without a possibility for risk-shifting is depicted in Fig. 2. The entire equity of the firm is held by its owners through a fund, which also has other securities in its portfolio. And this fund sells a capital insurance to the firm whose equity it holds. If the equity holders of the firm engaged in risk-shifting, they would not shift risk to the debt holders, but to the insurance providers - which means that they would shift the risk to themselves.

The capital insurance of a firm by a fund entails efficiency gains owing to the differences between a firm and a fund. First, being financed by issuing shares, a fund cannot become insolvent and there a no costs that result from an inefficient interruption of the fund operation in case of insolvency. Second, since (passive) funds are just a set of financial contracts that transmit payoffs (in contrast to firms, which create payoffs), it is common practice that funds are not subject to corporate taxation. Consequently, the provision of the insurance entails no losses. But it allows for gains, as it reduces the losses of the firm. Owing to these gains, the fund has an incentive to provide the capital insurance to the firm that it owns completely.

It is interesting that the firm-fund structure, which I have derived as a way to obtain efficiency gains, is effectively the same as the 'liability holding companies' (LHCs) that Admati et al. (2012) have suggested in the context of bank regulation. They propose LHCs with the aim to counteract negative incentives due to implicit bailout guarantees. Opponents of capital regulation argue that the choice of capital structure by banks would not be driven by such guarantees, but mainly by the trade-offs discussed above. The result of this paper is: if this is true, banks should actually welcome the establishment of LHCs, as they allow for private efficiency gains.

To sum up, this section has shown that there is simple way to create financial assets with a distribution of payoffs which allows for both, efficiency gains for the purchasing firm as well as a reduction of the insolvency risk. Having illustrated this for a single firm, a generalization of the result for a continuum of firms in a closed economy is given in Appendix B.[13] Given this strong and positive result, one might wonder why not all firms set up the firm-fund structure suggested here. Section 7 argues that the transition to such structures is inhibited by a problem of misaligned incentives that is similar to the 'leverage ratchet effect' highlighted by Admati et al. (2018).

6.1 A Capital Insurance in Presence of Rent Extracting Managers

The analysis of debt as disciplining device according to Diamond & Rajan (2000), which I have presented in Section 4, studied financial assets whose payoff R_S is independent of the managers of the firm that buys these assets. The last section has highlighted a capital insurance that yields $s\,R_S = \max\{0, D - R\}$ as simple way to create a financial asset with a beneficial payoff distribution. As mentioned in that section, it might be difficult to write a contract that specifies a payoff $\max\{0, D - R\}$ in each possible state, other than by condition on R itself. In that case, however, the payoff R_S becomes dependent on the behavior of the managers. And the disciplining effect of the demandable debt in states with $R \leq D$ gets lost, because the debt holder do not carry losses from the extraction by managers, but the capital insurance covers the loss. The payoff of the insurance increases with the reduction of R by a rent extraction by managers. Since the equity holders do not care for the rent extraction in states with $R \leq D$, the managers can thus increase the rent extraction in these states without any constraint.

There are (at least) two different solutions for this problem, depending on whether the depositors and the managers can collude. The solution for the case that they can collude

[13] Closed economy means that there are no externally provided financial assets, but all possible financial claims have to refer directly or indirectly to the payoff from the real assets of the firms.

is the more robust one. But let me also briefly point a possible solution for the case that they cannot collude.

In that case, a small modification of the capital insurance can solve the problem (if one follows the logic of Diamond & Rajan). Consider an insurance that does not only yield $D-R^n$, but $D-R^n+g(D-R^n)$ in every state with $R^n < D$, where R^n is the 'net payoff' of the firm, which means the payoff R from its assets minus the rent extraction by managers. And $\frac{d}{dx}g(x) < 0$ with $g(x) \geq 0$ for all $x \in [0, D]$. Furthermore, the additional payoff $g(D-R^n)$ in case of an insured event, which increases in R^n, shall accrue to the depositors who do not run. Running depositors simply receive their fraction of D. Given this kind of insurance contract, the debt holders maintain an incentive for monitoring. They can threaten the managers with a run, if their premium $g(D-R^n)$ decreases too strongly due to the rent extraction, which increases $D-R^n$. Since running depositors simply receive D, the value of keeping the managers in states with $R < D$ (when the insurance becomes effective) is $g(D-R)$. Bargaining over this continuation value, the depositors are in the same position which the theory of Diamond & Rajan assigns to equity holders. Assuming that equity holders depositors bargain in a similar way, the managers can obtain a fraction b_e of this continuation value. By choosing a function g with values slightly above zero, one can minimize the extraction $b_e\, g(D-R)$ of rents from the capital insurance. Moreover, with $b_e\, g(x)$ close to zero for all $x > 0$, the managers have no incentive to trigger an insured event by extracting so much that R^n falls below D, because the rent $b_e(R-D)$ that they can extract in states with $R > D$ is larger than $b_e\, g(x)$. As result, the capital insurance leads to an expected loss $\int_0^D b_e\, g(D-R)\, f(R)\, dR$. For sufficiently small g, however, this loss is smaller than the gains from preventing liquidations, which are $\int_0^D l\, Rf(R)\, dR$.

If the depositors and the managers can collude, however, this modified insurance contract cannot suppress the moral hazard, because the overall gains for managers plus depositors from exploiting the insurance (by extracting X) are larger than the costs: $\left|\frac{d}{dX}X\right| > \left|\frac{d}{dX}g(D-R-X)\right|$ for g close to zero. In that case, the modification g of the insurance payoff is useless and the insurance provider can simply provide the payoff $\max\{D-R, 0\}$. If the insurance provider also holds the firm equity, however, it still has a disciplining device owing to the power to replace the managers. As in states with $R > D$, the insurance provider (i.e., the equity holders) can bargain with the managers over the continuation value of keeping the managers in states with $D > R$. If the equity holders took over the firm, the resulting loss $l\,R$ from inefficient liquidations would increase the insurance payments that are necessary to pay out the depositors. The value of keeping the managers is thus the avoidance of this loss $l\,R$, which is equal to the loss that would occur in case of runs. The managers, however, can only obtain a fraction b_e of this value in the bargaining process. Consequently, the loss $b_e\, l\, R$ due to an extraction of rents from the capital insurance is smaller than the loss $l\,R$ that would occur in case of a run, which is prevented by the insurance. As a result, the capital insurance leads to efficiency gains, even if the managers can exploit this insurance and can collude with the depositors.

7 Obstacles to Integrated Funds

This paper shows that integrated funds allow for private gains for a firm, if its capital structure is chosen according to a trade-off between taxes and bankruptcy costs or a trade-off between rent extraction by managers and costs due to runs. If these trade-offs are empirically relevant, one should expect that all firms make use of integrated funds. But this is not the case. The reason might simply be that the trade-offs mentioned above are in fact not important for the choice of capital structure. But I want to suggest another explanation for the lack of integrated funds, which is related to the process of changing the capital structure.

In contrast to the assumption used in the analysis of the trade-offs theories, a firm usually has outstanding debt. In that case, a problem arises that has been highlighted by Admati et al. (2018) in their description of the 'leverage ratchet effect': If the face value of this outstanding debt cannot be renegotiated, the owners of the firm will not implement a change of capital structure that has a positive net present value owing to its reduction of expected bankruptcy or liquidation costs. The reason is the asymmetric distribution of gains and losses: the benefit of reduced bankrupty costs accrues to the debt holders, while the owners/equity holders incur the cost of higher taxes, for instance.[14]

If a firm could commit to the establishment of an integrated fund at a future point in time, the pricing of debt that is rolled over or newly issued could account for the reduction of the bankruptcy risk at this future point. As a consequence, the firm owners could participate in the gains from the integrated fund and would thus have an incentive to establish it in the long run. However, once the firm owners have incurred their part of the gains in the form of adjusted debt prices, they have an incentive to reduce the integrated fund or to choose its portfolio such that risk is shifted to the debt holders. Since there are so many degrees of freedom related to an investment in financial assets at a future point in time (as the set of available assets as well as their characteristics constantly evolve), it might be impossible to credibly commit to the future characteristics of an integrated fund. The consequence of this inability is that debt holders cannot fully trust in the safety of their claims and thus do not accept debt prices that account for prospective reductions in the insolvency probability and that allow to share the gains from integrated funds with the firm owners.

8 Implications for the Regulation of Banks

The results of this paper have important implications for the debate about the regulation of banks. There is the widespread notion that capital requirements for banks, which are

[14] Debt holders even gain at the expense of the equity holders in absence of such frictions, as highlighted by Admati et al. (2018). A reduction of the insolvency risk always implies that the payoff to holders of outstanding debt increases in some states. If the face value of their debt is not adjusted, but their debt contract is fixed, they gain at the expense of the equity holders.

intended to improve the stability of the financial sector, entail some costs. First, they are supposed to cause private costs for banks due to a deviation from their privately optimal choice of financing. And second, they are supposed to cause social costs - either indirectly, because the private costs for banks impair their provision of credit and other services to the economy, or directly, because the requirements allegedly reduce the volume of socially beneficial 'money-like' claims.

There are plausible arguments for private costs in the short run, when capital requirements are raised quickly. The increase in equity reduces the default probability of the outstanding debt and it thus transfers wealth from equity holders to the holders of outstanding debt, as described in Admati et al. (2016). And these private costs can lead to social costs, when the bank owners prefer to comply with increased capital requirements by liquidating assets or by forsaking new projects with positive NPV. The arguments for private and social costs of capital requirements in the long run, in contrast, are usually based on the trade-off theories discussed in this paper. This paper has shown, however, that these theories actually allow for a decrease of leverage and insolvency risk of banks without any costs, if one takes into account that banks can 'integrate a fund'. In fact, the integration of a fund in order to reduce bankruptcy risk can even provide gains.

Such beneficial reductions of the insolvency risk depend on the availability of assets with an appropriate distribution of payoffs. In Section 6, I have illustrated an example how financial assets with an appropriate distribution can be created. This example is depicted in Fig. 6 and it is effectively the same as the liability holding companies (LHCs) suggested by Admati et al. (2012). A regulation that takes LHCs into consideration could therefore reduce the insolvency risk of banks without any private costs in the long run, but rather with gains. In absence of private costs for banks, such regulation would also not entail any social costs, as indicated above. – To be precise, one type of private costs would actually arise: the loss of the subsidies that banks get from governments in form of implicit bailout guarantees. But as long as one does not want to subsidize banks in this way, one should not be concerned about this type of these costs.

Capital regulation based on integrated funds or LHCs faces a problem similar to the one discussed in the previous subsection: the regulation has to ensure that the size of the funds and their compositions are such, that the payoffs from the securities held in the funds are large enough in states in which the banks need them to avoid insolvency. As mentioned before, the banks might exploit a discretion about the fund portfolio for the purpose of risk-shifting. By imposing appropriate rules, however, the regulation can remove this discretion. This is a standard problem of capital regulation, which tries to alleviate risk-shifting at the expense of an explicit or implicit public insurance. It might be difficult to set rules that remove the discretion and the moral hazard completely. But this problem only affects the amount of implicit subsidies that banks can extract - it does not change the result that funds allow for a reduction of the insolvency risk of banks without efficiency losses in the long run.

Let me conclude with brief **estimates for the size integrated funds/LHCs**. I consider the case that the funds invest in relatively risky assets, namely corporate bonds, and I study their ability to absorb losses in financial crises. The weighted average of default rates of all corporate bonds rated by Moody's peaked at 8.424 % in 1933 and peaked again at 5.422 % in 2009.[15] One can thus expect that a fund which issues shares in order to purchase an amount X of bonds can provide a capital insurance worth $(1-\delta_D)X$ with $\delta_D = 0.1$ even in very bad states. This is a conservative estimate, since positive recovery rates are ignored and corporate bonds are a risky type of bonds.

Let us now consider a scenario in which the loss-absorbing capital of US banks shall be increased by 5% of their assets by means of LHCs that invest in bonds. This would more than double the amount of loss-absorbing capital in banks, given that they comply with the leverage ratio that is imposed by the current regulation, which is in the range of $3-5\%$. With a discount factor δ_D and an aggregate volume A_{agg} of bank assets, the volume V^D_{abs} of bonds that the funds would need to hold is $V^D_{abs} = \frac{1}{1-\delta_D} \cdot 0.05 \cdot A_{agg}$. Take the example of the US banks in December 2012:[16] According to the FDIC the aggregate volume of assets in insured US banks was $A_{agg} = \$\,14.5\,tn$.[17] This means that the LHCs would need to absorb bonds worth $V^D_{abs} = \$\,0.8\,tn$ in order to double their capital buffer.

To get an appropriate idea of this number, it should be compared to the volume of bonds available on the market. In case of the US market in December 2012, the volume of outstanding bonds was $\$\,36.6\,tn$ according to SIFMA.[18] Using information from Hanson et al. (2015) and the FDIC,[19] one can subtract the volume of bonds already held by banks. As a result, the volume of bonds that are not held by banks and that could be purchased by the related LHCs is at least $V^D_{ext} = \$\,33.8\,tn$. This means that only 2.4% of the available bonds would need to be purchased by LHCs in order to double the capital buffers of banks that are able to absorb losses even in the worst states of the economy.

[15] see `http://efinance.org.cn/cn/FEben/Corporate%20Default%20and%20Recovery%20Rates,1920-2010.pdf`

[16] a recent date for which all relevant data is easily accessible

[17] see `https://www.fdic.gov/bank/statistical/stats/2012dec/industry.pdf`

[18] see `http://www.sifma.org/uploadedFiles/Research/Statistics/StatisticsFiles/CM-US-Bond-Market-SIFMA.xls?n=13061`

[19] Hanson et al. (2015) state that 20.8 % of the assets of the banks in their sample were securities in 2012. Since bonds are only a part of this set, the given estimate for the volume of bonds already held by banks is an upper bound.

A Trade-off between Risk-Shifting and Effort Reduction

This appendix confirms the statements made in Section 5 by a formal analysis. More precisely, it shows that the possibility of costless decrease of leverage and insolvency risk by means of an integrated fund also holds in presence of agency costs that has been described in Jensen and Meckling (1976). This means that the section discusses a model with a trade-off between agency costs of debt in the form of risk-shifting and agency costs of equity in the form of a reduction in effort by the managers.[20] I present the analysis in two steps: first, the case of a firm without fund is established as a benchmark, before the impact of an integrated fund is illustrated.

A.1 Agency Costs of a Firm without an Integrated Fund

Consider again that a firm owner sells equity and debt claims and tries to maximize the revenue from this sale. The payoff of the firm assets at $t = 1$ depends on a basic cash flow R (with density f and upper bound $\overline{R}$) and on the behavior of the firm managers between $t = 0$ and $t = 1$:

1. The effort $c_m \in [0, \bar{c}_m]$ of the managers amplifies the payoff, so that it becomes $\rho(c_m) \cdot R$, with $\frac{d}{dc_m}\rho > 0$. Exerting the effort c_m, the managers incur a disutility that is equivalent to a negative payoff $-h(c_m)$ at $t = 1$, with $\frac{d}{dc_m}h > 0$. In order to incentivize the managers, the firm owner gives them a share $m \in [0, 1]$ of the firm equity at $t = 0$. (Later, I explain why the results also hold for a payment of managers with other claims.)

2. The managers can choose to increase the risk of a fraction $\alpha \in [0, 1]$ of the assets during the period. If upside of this change, which occurs with probability p, is that the asset payoff at $t = 1$ is raised to $\rho\, R + \alpha \cdot \beta^+$. The downside, occurring with probability $1 - p$, is that the payoff is reduced to $\rho R - \alpha \cdot \beta^-$.[21] Assume that the increase in risk is inefficient: $p\, \beta^+ < (1-p)\beta^-$.

The risk-neutral managers choose c_m and α between $t = 0$ and $t = 1$, after the firm has issued debt with face value D. Their optimization problem is then

$$\max_{c_m \in [0,\bar{c}_m], \alpha \in [0,1]} \left(m\, E_f \left[\max\left\{ 0\, ,\, \rho(c_m)R + \mathbf{1}_{\beta+}\alpha\beta^+ - (1-\mathbf{1}_{\beta+})\alpha\beta^- - D \right\}\right] - h(c_m) \right), \quad (6)$$

where $\mathbf{1}_{\beta+}$ identifies states with a positive outcome of the additional risk. The optimal choices c_m^* and α^* depend on m and D. The payoff of the firm at $t = 1$ is thus

$$X(R; D, m) := \rho\big(c_m^*(D, m)\big)R + \alpha^*(D, m) \cdot \left[\mathbf{1}_{\beta+}\beta^+ - (1 - \mathbf{1}_{\beta+})\beta^- \right] . \quad (7)$$

Assume that all agents have complete information at $t = 0$ and that the claims are again priced in markets with risk-neutral investors and riskfree rate $r = 0$. (Appendix C shows

[20] 'Effort' can also be interpreted as the discipline to abstain from a misuse of firm resources.

[21] In order to avoid uninformative case distinctions, assume that $\rho R - \beta^- > 0$ for all possible cases. One could allow for a dependence of β^- and β^+ on $\rho\, R$, but that would not change the results of this analysis.

that the results are robust to more general preferences of investors.) The value d of the debt at $t = 0$ is then $d(D, m) = E_f [\min\{D, X(R; D, m)\}]$. And the value of the equity at $t = 0$ is $e(D, m) = E_f [\max\{0, X(R; D, m) - D\}]$. The value v of the firm at $t=0$ is the sum of the values of debt and equity net of the equity given to the managers:

$$v(D, m) = E_f [X(R; D, m)] - m\, e(D, m)$$

The decision problem of the initial firm owner who wants to maximize the revenue from selling debt and equity is thus $\max_{D \in [0, \overline{R}],\, m \in [0,1]} v(D,\, m)$. The firm value $v(D, m)$ depends on $X(R; D, m)$, which depends on the behavior of the managers who choose their optimal $c_m^*(D, m)$ and $\alpha^*(D, m)$ according to Eq. (6). Choosing D and m at $t = 0$, the initial firm owner takes this dependence into account and trades off the agency cost of debt against the agency cost of equity.

Lemma 4
There is an optimal capital structure (D^*, m^*), which maximizes v.

Proof: The manager problem and the firm problem always have finite solutions, since both are optimizations of finite expressions over a compact set: for the manager problem given in Eq. (6), the choice set is $[0, \bar{c}_m] \times [0, 1]$ and the objective function is bounded from below by $-h(\bar{c}_m)$ and from above by

$$\begin{aligned} E_f \left[\rho(\bar{c}_m)R + \mathbf{1}_{\beta^+}\alpha\beta^+\big(\rho(\bar{c}_m)R\big)\right] &< E_f \left[\rho(\bar{c}_m)R + (1 - \mathbf{1}_{\beta^+})\alpha\beta^-\big(\rho(\bar{c}_m)R\big)\right] \\ &< 2\rho(\bar{c}_m)E_f [R] < \infty\,; \end{aligned} \tag{8}$$

and for the firm problem, the choice set is $[0, \overline{R}] \times [0, 1]$ and all terms in the objective function $v(D, m)$ are bounded from above and below, since this holds for $X(R; D, m)$ as implicitly shown in Eq. (8).[22]

Having a benchmark that represents the managers' impact on the firm assets and the trade-off between agency costs of equity and debt, let us now study the consequences of integrating a fund.

A.2 The Effect of an Integrated Fund

The possibility to integrate a fund means again that the firm can choose to invest an amount s at $t = 0$ in a set S of financial assets, which are offered in the same market in which the firm issues its debt and equity. As before, I study the firm problem for a fixed composition of the portfolio S that yields R_S at $t = 1$ per unit of s, and $\hat{f}$ denotes the joint distribution with R.

The behavior of the managers might be influenced by an integrated fund, such the optimal

[22]It is possible that several choices are equally optimal for the managers. Let us simply assume that managers choose each of these absolute maxima with equal probability in such cases.

choices c_m^* and α^* can depend on s. It seems to be reasonable, however, that the basic characteristics of the initial firm assets are not affected by financial assets held by the firm. I thus assume that the distribution f of the basic payoff R as well as the function ρ, which describes the effect of effort on the output of the initial firm assets, are independent of s. Let us also assume for a moment that β^+ and β^-, which means the potential increase of the risk of the initial firm assets, are independent of the purchased financial assets. In Sections 6 and 7, I have discussed how an integrated fund might expand the possibilities for risk-shifting. Given these assumptions, the payoff from the productive assets is

$$X(R;D,m,s) = \rho\big(c_m^*(D,m,s)\big)R + \alpha^*(D,m,s)\cdot\left[\mathbf{1}_{\beta^+}\,\beta^+ - (1-\mathbf{1}_{\beta^+})\beta^-\right]. \tag{9}$$

The impact of the integrated fund on the manager behavior (which means the form of $c_m^*(D,m,s)$ and $\alpha^*(D,m,s)$ as function of s) depends on the way in which the payment scheme of the managers is adjusted to the integration of a fund. While the firm has many degrees of freedom in choosing a scheme, I only present a simple example here, for which the integrated fund has neither a positive nor a negative effect on the firm value.
Let us consider the case that managers receive the fraction m of equity claims to the payoff X from the initial firm assets. If the payoff from the initial firm assets has priority (over the payoff from the purchased financial assets) in repaying the firm debt, the decision problem of the managers during the period is

$$\max_{c_m\in[0,\bar{c}_m],\alpha\in[0,1]}\left(m\,E_{\hat{f}}\left[\max\left\{0,\rho(c_m)R+\mathbf{1}_{\beta^+}\alpha\beta^+-(1-\mathbf{1}_{\beta^+})\alpha\beta^--D\right\}\right]-h(c_m)\right).$$

This problem is identical to the one in the benchmark case.

Observation 3
If the managers are paid with equity claims to the initial firm assets, then their behavior is independent of the integrated fund: $\alpha^*(D,m,s)=\alpha^*(D,m)$ and $c_m^*(D,m,s)=c_m^*(D,m)$. Consequently, the payoff from the initial firm assets is independent of the fund, too: $X(R;D,m,s)=X(R;D,m)$.

The key idea behind this incentive scheme is to relate the payment of the managers to the part of the firm that depends on their behavior. This is the set of initial firm assets whose payoff X depends on the managers' effort and their risk-shifting. The payoff R_S of purchased securities is independent of the managers of the firm that only holds the securities passively. The adjustment of the payment scheme is an example for the following, quite general point: there are important relations between the capital structure of a firm and the incentives of agents in that firm, but these relations are not as strict as first highlighted in Jensen & Meckling (1976). A firm has many degrees of freedom to shape these relations by writing better contracts with the involved agents. This has already been stressed by, for instance, Aghion & Bolton (1989), or in a similar case as this one, by Dybvig & Zender (1991) in their discussion of Myers & Majluf (1984).

The possibility to separate the manager behavior from the integrated fund is independent of the initial payment scheme of the managers. I have illustrated the case in which they only receive equity claims, but the same logic applies to any set of claims with which managers are paid. The structure and payoffs of their claims can be maintained when a fund is integrated, if they continue to refer to the initial firm assets.

Although the integrated fund does not change the behavior of the managers, it has an impact on the solvency of the firm. In states in which the payoff X from the firm assets is too small to repay the firm debt D, a sufficiently large payoff $s\,R_S$ from the fund can avoid insolvency. The insolvency probability ϕ is thus given as $\phi(D,m,s) = \int \mathbf{1}_{\{X(R;D,m,s)+s\,R_S\,<\,D\}}\,\hat{f}(R,R_S)\,dR\,dR_S$. And the integrated fund also affects the firm leverage, which is again defined as $l(D,m,s) = \frac{D}{E_{\hat{f}}[X(D,m,s)+s\,R_S]}$.

Observation 4

If the debt level D is kept fixed and the managers are paid with a share m of equity claims to the initial firm assets, then an increase in the size of an integrated fund leads to a decrease of both, the leverage and the insolvency probability:

$$\frac{d}{ds}l(D,m,s) < 0 \;\forall\; s \in \mathbb{R}^+, \qquad \frac{d}{ds}\phi(D,m,s) \leq 0 \;\forall\; s \in \mathbb{R}^+$$

and the second inequality is strict for some $s \in \mathbb{R}^+$ if $E_{\hat{f}}\left[\mathbf{1}_{\{R_S>0\}}\mathbf{1}_{\{X(R;D,m)<D\}}\right] > 0$.

The additional payoff $s\,R_S$ also affects the value of the debt claims at $t=0$, which becomes $d(D,m,s) = E_{\hat{f}}\left[\min\{X(R;D,m,s)+s\,R_S, D\}\right]$. The value e' of the equity of the overall firm (initial firm assets plus integrated fund) is equal to the value of the expected payoff from the initial firm assets and the integrated fund net of the expected debt payments and the expected payoff to the managers:

$$\begin{aligned} &e'(D,m,s) = E_{\hat{f}}\left[\max\{0, X(R;D,m,s)+s\,R_S-D\}\right] - m\,e_X(D,m,s)\,, \\ &\text{with}\;\; e_X(D,m,s) = E_{\hat{f}}\left[\max\left\{0, X(R;D,m,s)-D\right\}\right]\,. \end{aligned} \tag{10}$$

The value $v_s(D,m,s)$ of the firm with integrated fund is the joint value of d and e':

$$v_s(D,m,s) = E_{\hat{f}}[X](D,m,s) + s\,E_{\hat{f}}[R_S] - m\,e_X(D,m,s)\,.$$

The 'net firm value' v, which means v_s net of the value of the financial assets, is:

$$v(D,m,s) = v_s(D,m,s) - s\,E_{\hat{f}}[R_S] = E_{\hat{f}}[X](D,m,s) - m\,e_X(D,m,s)\,. \tag{11}$$

If the firm buys the financial assets in the same competitive market in which it issues its debt and equity, then **Assumption 2 a** applies again: the price of the financial assets at $t=0$ equals their expected payoff $E_{\hat{f}}[s\,R_S]$. The decision problem of the initial firm owner is then

$$\max_{D\in\mathbb{R}^+, m\in[0,1], s\in\mathbb{R}^+} \Big(v_s(D,m,s) - E_{\hat{f}}\left[s\,R_S\right]\Big) = \max_{D\in\mathbb{R}^+, m\in[0,1], s\in\mathbb{R}^+} v(D,m,s)\,. \tag{12}$$

Proposition 3

Consider a firm without integrated fund ($s \equiv 0$) whose optimal capital structure is (D_0, m_0). If this firm can integrate a fund and pays its managers with claims to the initial firm assets, then its optimal capital structure (D^*, m^*, s^*) is given by

$$D^* = D_0\,, \quad m^* = m_0\,, \quad \textit{and } s^* \textit{ being an abitrary element of } \mathbb{R}^+\,.$$

Consequently, an increase in the size of the integrated fund and a corresponding decrease of the firm leverage has no effect on the optimized net firm value:

$$v(D^*, m^*, s^*) = v(D_0, m_0, 0) \;\forall\; s^* \in \mathbb{R}^+.$$

The proposition follows directly from the fact that v is effectively independent of s, because $X(R; D, m, s)$ is effectively independent of s, when the manager payment remains aligned with the firm assets on which their behavior has an impact. The firm can thus increase its equity to any level without a reduction of its firm value.

To sum up, this section has shown that a key result of the previous sections also holds for the trade-off between agency costs of debt and equity: the integration of a fund allows for a decrease of leverage and insolvency risk without a loss of firm value. In contrast to the cases discussed before, this result does not depend on an appropriate payoff distribution of the financial assets, but on an appropriate payment scheme for the managers. Given the payment scheme discussed here, integrated funds do not increase the firm value, as in the previous sections, but they just maintain the value. Further research, however, might show that more refined payment schemes perhaps allow for an increase.

B Equilibrium

This appendix shows that the results obtained in the paper also hold on the aggregate level. This means that they hold in an economy with a finite set of firms and real assets, in which there are no externally provided financial assets, but all possible financial claims have to refer directly or indirectly to the payoff from the real assets of the firms. The section demonstrates that there is an equilibrium in which the possibility to integrate funds increases the net firm value of all firms in the economy and decreases the probability of firm insolvencies, simultaneously. This appendix thus generalizes the result obtained for the simple example in Section 2. In order to illustrate the effects of integrated funds, I first introduce a benchmark equilibrium with firms that can only invest in their real, productive assets, before I add the possibility of integrated funds.

B.1 The Equilibrium of the Benchmark Case Without Funds

At $t = 0$, there is a continuum of investors who buy claims to payoffs at $t = 1$. In accordance with the previous sections, I assume that all investors are risk-neutral. The financial market can consequently be characterized by the demand and supply of claims to expected payoffs at $t = 1$. The price for one unit of expected payoff is given by $\frac{1}{1+r}$ with r representing the riskfree interest rate. The demand and supply of claims, measured by the value of the claims at $t = 0$, shall be denoted by $\mathcal{I}^d$ and $\mathcal{I}^s$. Concerning $\mathcal{I}^d$, let us simply assume that the continuum of investors has an aggregate demand for financial claims which is continuous and monotonically increasing in r: $\mathcal{I}^d = \mathcal{I}^d(r)$ with $\frac{d}{dr}\mathcal{I}^d(r) > 0$ and $\mathcal{I}^d(-1) = 0$. These characteristics can be derived from saving-consumption-decisions of households, but the additional structure would not provide any further insights.
Assume that there is a continuum $J = [0, 1]$ of firms and each firm $j \in J$ maximizes its firm value v_j by choosing a vector of choice variables as described in the previous sections, with the temporary constraint of $s = 0$ (i.e., without integrated fund). The vector is (D_s, D_r) for the trade-off between taxes and debt benefits; it is (D) for the trade-off between liquidation losses and rent extraction; and it is (D, m) for the trade-off between agency costs of debt and equity. The optimally chosen vector of firm j shall be denoted as x_j. The expressions for the firm value at $t = 0$ can be easily generalized to any risk-free interest rates r, because the firm value is simply the sum of the $t = 0$-values of expected payoffs at $t = 1$. Accounting for the dependence of the discounting factor on r, the firm value $v(x_j; r)$ for $r \neq 0$ is given as $v(x_j; r) = \frac{1}{1+r}\, v(x_j)$ with $v(x_j)$ being the firm value for $r = 0$. Let us assume that the assets of the firms, which have been regarded as simply given in the previous analysis, require an initial investment of 1 at $t = 0$. The initial firm owner only invests in the assets and thus 'creates' the firm, if the investment has a strictly positive value at $t = 0$, which means if $v_j(x_j; r) - 1 > 0$; and it is inactive for $v_j(x_j; r) - 1 < 0$. For $v_j(x_j; r) = 1$, the owner is indifferent between being active or being

inactive. The aggregate supply $\mathcal{I}^s(r)$ of expected payoffs by the firms at $t = 0$ is:[23]

$$\mathcal{I}^s(r) = \int_J v_j(x_j; r)\mathbf{1}_{\left\{v_j(x_j;r)\geq 1\right\}}\, dj\,. \tag{13}$$

Observation 5

$\mathcal{I}^s(r)$ is continuous and monotonically decreasing in r with $\lim_{r\to\infty} \mathcal{I}^s(r) = 0$.[24]

As mentioned, $v_j(x_j; r)$ depends on r only through the discount factor $\frac{1}{1+r}$, which is continuous and monotonically decreasing in r. With a continuum of firms, these properties of $v_j(x_j; r)$ also apply to $\mathcal{I}^s(r)$.

Observation 6

There is a unique interest rate r^* for which the financial market clears with $\mathcal{I}^d(r^*) = \mathcal{I}^s(r^*)$.

The existence of a unique equilibrium follows directly from the continuity and monotonicity of supply and demand. Having established this benchmark case, the next subsection studies the effect of integrated funds on an aggregate level.

B.2 Equilibrium with Integrated Funds

The equilibrium of the benchmark case shall serve as reference point in this section. For that purpose, all parameters of the benchmark equilibrium are denoted by a subscript 0.

While Section 6 addresses the practical problem of creating financial assets with beneficial payoff distributions, let us impose a simplifying assumption here:

Assumption 3

There is a continuum $\mathcal{D} = [0, 1]$ of profit-maximizing, risk-neutral dealers with complete information at $t = 0$. They purchase debt and equity from the firms and sell derivatives (whose payoffs are conditional on the payoffs of the firms in the market) to firms and investors in perfect competition, while they have no own wealth at $t = 0$.

I assume that the cost of writing a simple derivative contract are negligibly small. The structure of the interdependent decision problems is as follows. For given r, the dealers, who anticipate the decision problems of the firms, demand equity and debt from the firms and offer financial assets to them. The firms solve their decision problems as described in the previous sections, including the possibility to integrate a fund by buying assets from the dealers. Given perfect competition, the dealers earn no profits and the prices of the financial assets equal their discounted expected payoffs.

[23] To be more precise, the supply function $\mathcal{I}^s(r)$ can be multi-valued, since the firm owners are indifferent about being active or inactive for $v_j(x_j; r) = r$. Consequently, $\mathcal{I}^s(r)$ maps to all values in the interval between $\int_J v_j(x_j; r)\mathbf{1}_{\left\{v_j(x_j;r)>1\right\}}\, dj$ and $\int_J v_j(x_j; r)\mathbf{1}_{\left\{v_j(x_j;r)\geq 1\right\}}\, dj$.

[24] As mentioned in Footnote 23, $\mathcal{I}^s(r)$ might be multi-valued at some r. It is yet continuous at these points in the sense of multi-valued functions, which means it is upper-hemicontinuous as well as lower-hemicontinuous.

The demand for financial assets by the firms depends on the capital structure theory that describes v_j. If agency costs determine the optimal capital structure and the firm chooses the payment scheme that has been discussed in A.2, then a firm is indifferent about the integration of a fund. If the trade-off theories discussed in Section 3 or 4 apply, then an unconstrained firm will demand a combination of financial assets that add up to a complete hedge of the payoff of its productive assets. Let us focus on this case for the remainder of this section. In order to simplify the discussion, let us impose:

Assumption 4
The payoff R_j of the productive assets of each firm $j \in J$ has a strictly positive and finite lower bound $\underline{R}_j$ as well as a positive and finite upper bound $\overline{R}_j$.

The purpose of the assumption is mainly to ensure that there is a strictly positive minimal payoff in each possible state. If this holds, it is feasible that all firms in the economy integrate the optimal set of financial assets (which amounts to a complete hedge), as we will see in the following. In principle, there are infinitely many ways how the competitive dealers buy claims from firms and offer financial assets to them, which all add up to an optimal set of financial contracts. An optimal set of financial contracts means that it reduces the costs from frictions within the firms to zero, so that no additional financial asset can improve the net firm value any further. For simplicity, I illustrate such optimal sets of contracts by a particular example with two large dealers, denoted as $\mathcal{D}_1$ and $\mathcal{D}_2$, which represent subsets of the competitive dealers.
Consider the case that the dealer $\mathcal{D}_1$ buys the fraction $\frac{\underline{R}_j}{\overline{R}_j}$ of the debt issued by all firms $j \in J^+ := (\frac{1}{2}, 1] \subset J$. This investment yields a nonvanishing payoff in each possible state, which allows to engage in the following operations. Each firm $j \in J^- := [0, \frac{1}{2}] \subset J$ optimally chooses $D_j = \overline{R}_j$ and demands a set of financial assets that yields $\overline{R}_j - R_j$ in each state. This choice reduces the tax payments/rent extraction to zero (since there is no equity payoff), while it also reduces the costs from bankruptcies/liquidations to zero (since the firm always remains solvent owing to $R_j + (\overline{R}_j - R_j) = \overline{R}_j = D_j$). Note that this choice implies a weak increase of the debt level relative to the benchmark case in which $D_j \leq \overline{R}_j$ holds.[25] Because each single firm $j \in J^-$ is infinitesimally small relative to the aggregate payoff that $\mathcal{D}_1$ receives from its fraction of the debt of firms in J^+, it is feasible that $\mathcal{D}_1$ offers the hedge demanded by a single firm $j \in J^-$. Consider that $\mathcal{D}_1$ does not only offer the hedge to this firm, but that it also buys the fraction $1 - \frac{\underline{R}_j}{\overline{R}_j}$ of the debt of this firm (which yields $\overline{R}_j \cdot \left(1 - \frac{\underline{R}_j}{\overline{R}_j}\right) = \overline{R}_j - \underline{R}_j > \overline{R}_j - R_j$ in each state). The two-sided deal (providing $\overline{R}_j - R_j$ and buying the fraction $1 - \frac{\underline{R}_j}{\overline{R}_j}$ of its debt) does not decrease the payoff that the dealer can sell to other agents. Basically, the dealer provides a payoff that 'flows throw the firm' and reduces the frictions therein, before the dealer 'collects' it again, in addition to a fraction of the payoff from the productive assets of that firm.

[25] The debt level $D_j = \overline{R}_j$ is highest meaningful debt level of a firm without fund, because $\overline{R}_j$ is the highest possible payoff of the assets, and any $D_j > \overline{R}_j$ is equivalent to choosing $D_j = \overline{R}_j$.

Since there is no loss of payoff by this two-sided deal, the dealer can offer it to all firms in J^-. And these firms demand it, since it allows for a reduction of their frictions to zero. As a consequence of these two-sided deals with the firms in J^-, the dealer $\mathcal{D}_1$ collects a large part of the payoff from their productive real assets. It can finance the purchase of this part of the payoff by selling claims to investors. Basically, the dealer acts like a investment fund that purchase debt claims from many different firms and, in addition, sells hedges to them. As mentioned, I assume perfect competition between the dealers, so that $\mathcal{D}_1$ earns no profits and the purchased and sold state-contingent payoffs net out in the aggregate. The gains from the reduction of the frictions within the firms accrue to the firm owners and the external investors, as we will see below.

The example is completed by the second set of firms and the second dealer $\mathcal{D}_2$. It holds the share $\frac{\underline{\underline{R}}_j}{\overline{R}_j}$ of the debt of firms $j \in J^-$, which provides a nonvanishing payoff in each possible state. This allows to engage in two-sided deals with firms in J^+, as they have been described above (i.e., selling a hedge plus purchasing the fraction $1 - \frac{\underline{\underline{R}}_j}{\overline{R}_j}$ of debt). As a result, all firms in J are completely hedged, choose maximal debt financing, and are able to avoid all costs that are due to the frictions described in the Sections 3 and 4. In particular, this implies:

Observation 7

Integrated funds allow to reduce the insolvency risk of all firms in the economy to zero, although the real assets remain the same and the debt level rather increases than decreases.

Let us now study the aggregate supply and demand of financial assets that results from these optimal choices of firms and dealers. The aggregate demand and supply, measured in terms of the value of the claims at $t = 0$, shall be denoted as $\mathcal{I}^d$ and $\mathcal{I}^s$, again. The supply of claims by an active firm $j \in J$, which chooses to integrate a fund with size s_j, equals $(v_j + s_j)$. The aggregate supply of financial assets by the dealers shall be denoted as $\mathcal{I}^s_D(r)$. The overall supply of financial assets at $t = 0$ is thus

$$\mathcal{I}^s(r) = \mathcal{I}^s_D(r) + \int_J (v_j(x_j^S\,;\, r) + s_j)\, \mathbf{1}_{\left\{v_j(x_j^S\,;\,r)\geq 1\right\}}\, dj\,, \tag{14}$$

where x_j^S denotes the optimally chosen vector of variables in the firm problem that allows for an unconstrained choice of the integrated funds. The demand for financial claims by the investors is the same as in the benchmark case, and shall be denoted as $\mathcal{I}^d_{inv}(r)$ here. In addition, there is the aggregate demand of the dealers, which shall be denoted $\mathcal{I}^d_D(r)$. And each active firm $j \in J$ demands the amount s_j of financial assets. The total demand is therefore

$$\mathcal{I}^d(r) = \mathcal{I}^d_{inv}(r) + \mathcal{I}^d_D(r) + \int_J s_j\, \mathbf{1}_{\left\{v_j(x_j^S\,;\,r)\geq 1\right\}}\, dj\,. \tag{15}$$

It is useful to distinguish between the gross supply and demand stated in the Eqs. (14) and (15) and the net supply and demand, $\mathcal{I}^{s,n}$ and $\mathcal{I}^{d,n}$, in which the claims held between firms and dealers are netted out. The net supply represents the volume of expected payoffs by the productive firm assets, and the net demand represents the volume of financial claims held by external investors. Since the dealers are unable to earn profits in perfect competition, the value of the financial assets that they offer equals the value of the securities that they hold: $\mathcal{I}_D^d(r) = \mathcal{I}_D^s(r)$. Furthermore, the value of the financial assets demanded by the firms is equal to the funding they need to buy them ($s_j = s_j$). Consequently, the net demand and supply of claims are given as

$$\mathcal{I}^{s,n}(r) := \mathcal{I}^s(r) - \mathcal{I}_D^d(r) - \int_J s_j \, \mathbf{1}_{\left\{v_j(x_j^S\,;\,r)\geq 1\right\}} \, dj \;=\; \int_J v_j(x_j^S\,;\,r) \mathbf{1}_{\left\{v_j(x_j^S\,;\,r)\geq 1\right\}} \, dj \tag{16}$$

$$\mathcal{I}^{d,n}(r) := \mathcal{I}^d(r) - \mathcal{I}_D^s(r) - \int_J s_j \, \mathbf{1}_{\left\{v_j(x_j^S\,;\,r)\geq 1\right\}} \, dj \;=\; \mathcal{I}_{inv}^d(r) \tag{17}$$

The net firm value is unaffected by integrated funds in case of the trade-off between agency costs, as it has been described in Appendix A. But if the trade-offs described in the Sections 3 and 4 apply, the integration of a fund increases the net firm value v_j of firms that have a strictly positive bankruptcy probability in the benchmark case. By buying the appropriate assets provided by the dealers, these firms can reduce the expected bankruptcy/liquidation costs and can raise their value. Consequently, there can be firms in J which are inactive in the benchmark equilibrium, but which are able to raise their net firm value v_j above 1 owing to the possibility to integrate a fund. If this is true for a non-vanishing mass of firms, the supply $\mathcal{I}^s(r)$ as well as the net supply $\mathcal{I}^{s,n}(r)$ of financial claims increase relative the benchmark case.

Observation 8

The net supply $\mathcal{I}^{s,n}(r)$ is continuous and monotonically decreasing in r, and it is weakly larger than in the benchmark case without integrated funds (described in Eq. (13)):

$$\mathcal{I}^{s,n}(r) \geq \mathcal{I}_0^s(r) \text{ for all } r > 0\,.$$

Observation 9

There is a unique market-clearing interest rate r^* with $\mathcal{I}^d(r^*) = \mathcal{I}^s(r^*) \wedge \mathcal{I}^{d,n}(r^*) = \mathcal{I}^{s,n}(r^*)$. This rate (which is the expected payoff at $t = 1$ per unit of claim sold to investors at $t = 0$) as well as the aggregate net volume $\mathcal{I}^{s,n}(r^*)$ of claims that firm owners can sell are weakly larger than in the benchmark equilibrium: $r^* \geq r_0$ and $\mathcal{I}^{s,n}(r^*) \geq \mathcal{I}_0^s(r_0)$.

While integrated funds weakly increase the net supply of expected payoffs, the net demand by investors is the same as in the benchmark case. As a consequence, the equilibrium interest rate (which is inversely related to the price) as well as the net supply in equilibrium weakly increase relative to the benchmark case. To sum up, this section has shown that there is an equilibrium in which all firms can simultaneously reduce their bankruptcy risk and increase their firm value owing to integrated funds, although the set of real, productive assets is fixed and although the debt level of the firms do not decrease.

C Generalized Preferences of Investors

In order to analyze the robustness of the results to generalized preferences of investors, let us study the same models as in the Sections 3 & 4 and in Appendix A, but consider an alternative pricing of the debt and equity. Let Σ denote the set of all possible states at $t = 1$, in which the assets yield state-contingent payoffs $R(\sigma)$ and $R_S(\sigma)$. To simplify the discussion, let us assume that $\hat{f}(x,y) := \int_\Sigma \mathbf{1}_{\{R(\sigma)=x\}} \mathbf{1}_{\{R_S(\sigma)=y\}} d\sigma$ is continuous in x and y. Assume furthermore that the equity and debt claims issued by the firms can be held by investors through a series of funds provided in a perfectly competitive financial market without entry or contracting costs. Consequently, these funds earn zero profits and are structured such that the diverse preferences of the investors are satisfied optimally. This implies that the prices of debt and equity claims are given by their decomposition into Arrow-Debreu securities and by the prices $p(\sigma)$ of these securities at $t = 0$, with $0 \leq p(\sigma) < \infty$. See Hellwig (1981) for a more detailed discussion of such decompositions of financial claims into state-contingent securities. The assumption of perfect capital markets does not contradict the purpose of this paper, which is the analysis of optimal capital structures on the firm level. The paper critically discusses trade-off theories that deviate from the Modigliani-Miller Theorem because of frictions within firms, not because of frictions within the capital markets.

Let us now study the value of a firm for this generalized pricing of payoffs, and let us start with the **trade-off between taxes, bankruptcy costs and a premium for safe debt**. Since all steps in the derivation of the firm value remain the same, apart from the pricing kernel, the expressions in Eqs. (1) and (2) simply become

$$\begin{aligned}
&v_s(D_r, D_s, s) = \\
&\quad \lambda\, D_s + \int_\Sigma \Big(R(\sigma) + s\, R_S(\sigma) - T\big(R(\sigma) + sR_S(\sigma) - D\big) - b\, \mathbf{1}_{\{R(\sigma)\,+\,s\,R_S(\sigma) < D\}} \Big) p(\sigma)\, d\sigma\,, \\
&v(D_r, D_s, s) = v_s(D_r, D_s, s) - \int_\Sigma s\, R_S(\sigma) p(\sigma)\, d\sigma \\
&\qquad\qquad = \lambda\, D_s + \int_\Sigma \Big(R(\sigma) - T\big(R(\sigma) + sR_S(\sigma) - D\big) - b\, \mathbf{1}_{\{R(\sigma)\,+\,s\,R_S(\sigma) < D\}} \Big) p(\sigma)\, d\sigma\,.
\end{aligned}$$

with $D = D_r + D_s$. The utility that investors incur from safe debt and the corresponding premium λD_s are not state-contingent, and the premium is thus accounted as separate term. The effect of the integration of a fund is analogous to the risk-neutral case: the fund increases the equity payoffs that are taxed, but it reduces the risk of insolvency and it might increase the level of safe debt that can be issued. The result stated in Proposition 1 thus remains valid, if one accounts for the generalized pricing. This means that the condition stated in Eq. (3) becomes

$$\lim_{s\to 0} \int_\Sigma b\, \mathbf{1}_{\{R(\sigma) < D_0\}}\, \mathbf{1}_{\{R(\sigma) + s\, R_S(\sigma) \geq D_0\}}\, p(\sigma)\, d\sigma > \int_\Sigma R_S(\sigma)\, T'\big(R(\sigma) + s\, R_S(\sigma) - D_0\big)\, p(\sigma)\, d\sigma.$$

And the condition in Eq. (4) becomes

$$\lim_{s\to 0}\int_\Sigma b\,\mathbf{1}_{\{R(\sigma)<D_0\}}\mathbf{1}_{\{R(\sigma)+sR_S(\sigma)\geq D_0\}}\,p(\sigma)d\sigma+\lambda\,\min\big(R_S|R{=}\underline{R}\big)$$
$$>\int_\Sigma R_S(\sigma)\,T'\big(R(\sigma)+sR_S(\sigma){-}D_0\big)p(\sigma)d\sigma\,,$$

with $\underline{R}$ being the lower bound for R: $\underline{R}=\min\big(R(\sigma)|\sigma\in\Sigma\big)$. The implications of this result are completely analogous to the case with risk-neutral pricing. The integration of a fund can both, reduce the insolvency risk and increases the firm value. And for each set of firm assets and corresponding optimal capital structure with positive insolvency probability, there exist financial assets with a payoff distribution which fulfills the conditions stated above. This is illustrated by the example of an asset that yields $R_S(\sigma)=\frac{1}{m}$ for all $\sigma\in\Sigma$ with $R(\sigma)=[0,D_0)$ and zero in all other states, where m is a normalization parameter.

The results for the two other specifications of the model can be generalized in the same way. In case of the **trade-off between rent extraction and liquidation losses**, the problem of the firm owner in presence of state-contingent pricing is $\min_{D\in\mathbb{R}^+,s\in\mathbb{R}^+}L(D,s)$, with [26]

$$L(D,s)=\int_\Sigma b_e\,\big(l\,R+l_S\,sR_S\big)\,\mathbf{1}_{\{D\leq R_l\}}\,p(\sigma)\,d\sigma+\int_\Sigma b_e\,\big(R{+}sR_S{-}D\big)\,\mathbf{1}_{\{R_l\leq D\leq R+sR_S\}}\,p(\sigma)d\sigma$$
$$+\int_\Sigma\big(l\,R+l_S\,sR_S\big)\,\mathbf{1}_{\{R+sR_S\leq D\}}\,p(\sigma)\,d\sigma\,,$$

and all payoffs of the assets are state-contingent: $R=R(\sigma)$, $R_S=R_S(\sigma)$, $R_l=R_l(\sigma)$. Proposition 2 remains valid for generalized preferences, if Eq. (5) is replaced by:

$$\lim_{s\to 0}\int_\Sigma l\,R\,\mathbf{1}_{\{R<D_0\}}\,\mathbf{1}_{\{R+s\,R_S\geq D_0\}}\,p(\sigma)\,d\sigma$$
$$\geq\int b_e\,R_S\,\mathbf{1}_{\{(1-l)R\leq D\leq R\}}\,p(\sigma)d\sigma+l_S\,\bigg(\int R_S\,\mathbf{1}_{\{R\leq D\}}\,p(\sigma)\,d\sigma+\int b_e\,R_S\,\mathbf{1}_{\{D\leq(1-l)R\}}\,p(\sigma)\,d\sigma\bigg).$$

Again, for each set of firm assets and corresponding optimal capital structure with positive insolvency risk, there is a possibility to simultaneously decrease the insolvency risk and to increase the firm value by means of an integrated fund.

Finally, in case of a **trade-off between agency costs of debt and equity** (as described in Appendix A), the robustness of the results with respect to generalized preferences of the investors is straight-forward. If the firm has chosen an optimal capital structure given its firm-specific assets and has aligned the payment scheme/the incentives of the managers with the firm production, then the integration of a fund has no effect on the behavior of the managers, independent of the pricing of the state-contingent payoffs. If the fund is

[26] For simplicity, the bargaining game (i.e., the parameter b_e) is assumed to be independent of the state-contingent preferences of the agents.

integrated without an increase of the debt level, the insolvency risk of the firm decreases.

D Proofs

D.1 Proposition 1

The derivative of the net firm value $v(D_r, D_s, s)$ w.r.t. s is

$$\frac{d}{ds} v(D_r, D_s, s) = -\int R_S\, T'(R + s\, R_S - D_r - D_s) \hat{f}(R, R_S)\, dR\, dR_S - b\, \frac{d}{ds} \phi(D_r, D_s, s)$$

With $D = D_r + D_s$, the derivative of the bankruptcy probability is:

$$\begin{aligned} \frac{d}{ds} \phi(D_r, D_s, s) &= \frac{d}{ds} \int_0^D \int_0^{\frac{1}{s}(D-R)} \hat{f}(R, R_S)\, dR\, dR_S \\ &= -\int_0^D \frac{1}{s^2}(D-R)\, \hat{f}\left(R, \frac{1}{s}(D-R)\right) dR = \int_{\frac{1}{s}D}^{0} R' \hat{f}(D - sR', R') dR' \\ \lim_{s \to 0} \frac{d}{ds} \phi(D_r, D_s, s) &= -\int_0^\infty R' \hat{f}(D, R') dR' = -f(D)\, E_{\hat{f}}\left[R_S | R = D\right] \end{aligned}$$

Plugging the derivative of ϕ into the derivative of v and evaluating it at $(D_s = D_{s,0}, D_r = D_{r,0}, s = 0)$, one finds that $\frac{d}{ds} v(D_{s,0}, D_{r,0}, s)\big|_{s=0} \geq 0$, if

$$b\ E_{\hat{f}}\left[R_S | R = D_0\right]\ f(D_0) - \int R_S\, T'(R - D_0)\, \hat{f}(R, R_S)\, dR\, dR_S \geq 0\,.$$

While the bankruptcy probability does not depend on the composition of D, safe debt earns a premium λ. Consequently, the firm always chooses the highest possible value for D_s, which is the lowest possible realization of $R + s\, R_S$. The derivative of this value w.r.t. s evaluated at $s = 0$ is $\min\left(R_S | R = \underline{R}\right)$. Accounting for this increase in the level of safe debt and the related premium, one has $\frac{d}{ds} v(D_s(s, D_0), D_r(s, D_0), s)\big|_{s=0} \geq 0$, if

$$b\ E_{\hat{f}}\left[R_S | R = D_0\right]\ f(D_0) + \lambda\, \min\left(R_S | R = \underline{R}\right) - \int R_S\, T'(R - D_0)\, \hat{f}(R, R_S)\, dR\, dR_S \geq 0\,.$$

D.2 Proposition 2

Computing the derivative $\frac{d}{ds} v(D, s) = -\frac{d}{ds} L(D, s)$ yields:

$$\begin{aligned} \frac{d}{ds} L(D, s) = &-\int_0^D \frac{D-R}{s^2} \left(l\, R + l_S\, (D-R)\right)\, \hat{f}\left(R, \frac{D-R}{s}\right)\, dR \\ &+ \int_0^D \int_0^{\frac{1}{s}(D-R)} l_S\, R_S\, \hat{f}(R, R_S) dR_S dR + \int_0^{\frac{D}{1-l}} \int_{\frac{1}{s}(D-R)}^{\frac{D-(1-l)R}{s(1-l_S)}} b_e\, R_S\, \hat{f}(R, R_S) dR_S dR \\ &+ \int_0^\infty \int_{\frac{D-(1-l)R}{s(1-l_S)}}^\infty b_e\, l_S\, R_S\ \hat{f}(R, R_S)\ dR_S dR \end{aligned}$$

Terms that cancel out are not displayed. Applying the same substitution of the integration variable as in the proof of Proposition 1, one can write the derivative $\frac{d}{ds}L$ for $\lim_{s\to 0}$ as

$$
\begin{aligned}
&-\int_0^\infty R'\, l\, D\, \hat{f}(D,R')\, dR' + \int_0^D \int_0^\infty l_S\, R_S\ \hat{f}(R,R_S)\ dR_S dR \\
&\quad + \int_D^{\frac{D}{1-l}} \int_0^\infty b_e\, R_S\ \hat{f}(R,R_S)\ dR_S dR + \int_{\frac{D}{1-l}}^\infty \int_0^\infty b_e\, l_S\, R_S\ \hat{f}(R,R_S)\ dR_S dR \\
&= -l\, D\, E_{\hat{f}}\left[R_S | R{=}D\right] \cdot f(D)\ + \int_0^D l_S\, E_{\hat{f}}\left[R_S|R\right] \cdot f(R)\, dR \\
&\quad + \int_D^{\frac{D}{1-l}} b_e\, E_{\hat{f}}\left[R_S|R\right] \cdot f(R)\, dR + \int_{\frac{D}{1-l}}^\infty b_e\, l_S\, E_{\hat{f}}\left[R_S|R\right] \cdot f(R)\, dR
\end{aligned}
$$

The derivative of $v(D,s)$ w.r.t. s is positive at $s = 0$, if this expression is negative. The statement in Proposition 2 is given by comparing the negative first term with the remaining positive terms for both cases.

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