Is a uniform price on Carbon desirable? A public finance perspective*

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Abstract

Should climate policy rely on a price of Carbon that is uniform across sectors? This paper studies this question from a public finance perspective. It is found that a justification for a uniform price can be given, but it relies on strong assumptions, among them indifference with respect to the distributive consequences of climate policy. Distributive considerations may imply that sectors whose output is consumed mostly by "the poor" should contribute less to meeting the government's emission target, whereas sectors whose output is consumed mostly by "the rich" should contribute more.

Keywords: Climate policy, equity-efficiency trade-off, optimal taxation.

JEL codes: D63, H21, H22, Q58

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1 Introduction

Actual climate policy is a mix of sector specific green taxes, sector-specific cap-and-trade-systems and sector specific regulation.[1] For the proponents of a uniform price on Carbon, this plethora of sector-specific rules appears as a political failure, an inability of the political process to reach climate policy targets in an efficient way. Against this background, this paper studies the desirability of a uniform price on Carbon drawing on the theory of taxation.

Foundational work in public finance offers various broad lessons. The production efficiency theorem by Diamond and Mirrlees (1971) gives conditions under which prices of inputs should remain undistorted. Can emission permits be viewed as an input for production and does this imply that a uniform price of Carbon is desirable? Contributions to the theory of taxation in the tradition of Ramsey (1927) yield rules for sector-specific taxes on consumption goods. These are derived under the assumption that there is an exogenous revenue requirement of the government. When the government has an emission target rather than a revenue target, does this imply that consumption goods should be taxed differentially depending on their Carbon footprint? The literature on the optimal mix of consumption and income taxes in the tradition of Atkinson and Stiglitz (1976) provides conditions under which distributive considerations are separable from the design of commodity taxes and should be dealt with only by the tax and transfer system. Does this imply that distributive considerations are orthogonal to the pricing of Carbon? The literature on tax incidence following Harberger (1962) shows that taxes have distributive implications for producers and consumers which depend on the relative elasticities of demand and supply. Do tradeoffs between the welfare of consumers and the welfare of producers matter for the desirability of a uniform Carbon price? This paper provides answers to these questions, while drawing on and extending these strands of the literature in public finance.[2]

Specifically, we analyze a model in which individuals differ in their incomes and in their preferences for green versus brown consumption goods. Firms' incentives to reduce the emission intensity of their production depend on the Carbon prices they are facing. A government has various policy instruments at its disposal that include sector-specific emission prices and output taxes. The government is, moreover, constraint by the need to reach a target level of emissions. For this setup, we clarify that there are restrictive conditions under which the insistence on a uniform price of Carbon is fully justified in the sense that any departure from it implies a violation of Pareto-efficiency. We then

[1]To give examples, the European Union emission trading system (EU ETS) covers electricity and heat generation, aluminium, cement, and steel works. The EU is planning to introduce a separate emission trading system (EU ETS II) covering buildings and road transport. The EU moreover has CO2 emission performance standards for cars and vans. In addition there are measures at the national level. Germany, for instance, has green taxes covering fossil fuels and electricity. One could go on.

[2]Section 2 contains a detailed account of recent contributions to these strands of the literature.

move away from these special cases and provide a more general treatment. We first evaluate a uniform price of Carbon under an assumption of *distributive indifference*. Under this assumption, reaching a given emission target in an efficient way is the only goal of climate policy. We provide sufficient conditions for the desirability and also for the non-desirability of a uniform Carbon price under distributive indifference. We show, moreover, that sector specific rules and hence a departure from a uniform price can be desirable with concerns for the distributive consequences of climate policy. A broad lesson from the analysis is this: Both a uniform Carbon price and a sector-specific approach are justifiable under appropriate conditions and in both cases the justification requires an explicit value judgment regarding the distributive implications of climate policy.

A sketch of the model. We consider an economy that has three sectors, one produces an unspecific consumption good, one produces a green good and one produces a brown good. The brown and the green good are imperfect substitutes. A possible interpretation is that next to the unspecific consumption good, there is a second good (e.g. "mobility") which is generated with a mix of brown alternatives (airline travel, commuting by car) and green alternatives (railway travel, public transport). We think of the brown and the green sector as potential targets for sector specific policies vis à vis "the rest of the economy". In all sectors, price-taking firms operate with decreasing returns to scale and realize inframarginal profits. Their production comes with emissions and they can exert abatement effort to reduce the emission intensity of their production. Individuals receive labour income and differ in productive abilities, as in Mirrleesian models of income taxation, and there is a non-linear tax on labour incomes. In addition, individuals differ in their willingness to pay for green consumption goods. Thus, there is a joint distribution of incomes and preferences for green goods. Individuals also own the firms in the economy and differ in their share in the economy's total profits. Some individuals mostly have labour income, others mostly have "capital income". Possibly, individuals also differ in their portfolios, e.g. with some individuals having larger stakes in the brown sector, and others having larger stakes in the green sector. The government's policy choices are constraint by the need to reach an economy-wide emission target. To reach the target, firms in the unspecific sector, the green sector and the brown sector need to reduce emissions. The analysis focusses on the question whether it is optimal to reach the emission target with sector-specific rules such as sector-specific emission targets or sector specific taxes. Alternatively, there is a price on emissions that is uniform across sectors and, if anything, a uniform consumption tax. If that's the case, we say that there is a uniform price of Carbon.

The differences in the sources of income, in productive abilities and in the preferences for brown versus green consumption goods imply that alternative policies that reach the government's emission target differ in their distributive implications. A central feature of the model is the possibility of a conflict between "green redistribution" and "inequalityreducing redistribution." Individuals with a strong taste for the green consumption good have a smaller carbon footprint per dollar spent on consumption goods than individuals with a weak taste for the green good. Therefore, a hypothetical redistribution of one unit of income from the former to the latter would reduce the demand for emission-intensive goods. If the preferences for green rather than brown goods are, moreover, positively correlated with incomes than such redistribution tends to increase the disposable income of "the rich" at the expense of the disposable income of "the poor."

As clarified in the Appendix, there are two special cases of this environment in which a uniform Carbon price is clearly desirable in the sense that any departure from it implies a violation of Pareto-efficiency. In the first special case, all individual characteristics – i.e. preferences for brown versus green consumption goods, productive abilities and "capital incomes" – are observable. In the second special case, productive abilities are the only source of heterogeneity amongst individuals and, moreover, assumed to be private information. Hence, in the second case, all individuals are assumed to have the same consumption preferences and to receive the same "capital incomes." These benchmark results are derived from a primal approach: Allocations are chosen so as to minimize overall emissions subject to the requirements (i) of physical feasibility, (ii) to reach a predetermined profile of utilities and (iii) of incentive compatibility when productive abilities are taken to be private information.

The main text uses a dual approach, however. It presents an analysis of how market outcomes change when climate policy deviates from a uniform price on Carbon. Sector-specific taxes drive a wedge between consumer and producer prices and there are numerous general equilibrium effects. For instance, labour incomes depend not just on the properties of the income tax, but also on consumer prices which change in response to changes of emission or commodity taxes. Also, a tax that increases consumer prices in one sector shifts the excess demand curves in other sectors with repercussions for the whole vector of equilibrium prices. Thereby it also affects the firms' profits and tax revenue.

The test for the desirability of a uniform carbon price then proceeds as follows: We consider a competitive equilibrium that results under uniform commodity taxation and with a price for emissions that is uniform across sectors. We then employ a generic social welfare function to evaluate deviations from this policy. Admissible deviations are those that respect the government's emission target. The welfare implications of an admissible deviation are captured by a sufficient statistics formula that highlights efficiency losses from sector-specific taxes, but also distributive effects across individuals with different consumption preferences or different sources and levels of "capital income." The extent to which these distributive effects are desirable then depends inter alia on the specification of welfare weights. When there are no welfare gains from a deviation, we say that a uniform price of Carbon is desirable.

The main result. We show that a uniform price on Carbon is desirable when two conditions are fulfilled. First, a condition of *distributive indifference*. It holds when the welfare weights are the same across all individuals, irrespectively of whether their "capital" or labour incomes are high or low, and irrespective of whether or not they spend much of their disposable income on green or brown goods. Second, a condition of *proportional fiscal externalities*[3] which holds when, for all the policy instruments that we study, their impact on income tax revenue is proportional to impact on the revenue from carbon pricing. Note that these conditions are different in nature. *Distributive indifference* is an explicit value judgment. *Proportional fiscal externalities* is an assumption on how the economy works. If both conditions are fulfilled, a uniform price on Carbon is justified. When the second condition is not satisfied, a second-best logic implies that a move away from uniform Carbon pricing is desirable even under *distributive indifference*.

Together the two conditions are sufficient for a justification of a uniform price of Carbon. This leads to the question whether they are necessary. The answer is "no". First, as explained above, when individuals differ only in productive abilities, a uniform Carbon price is desirable even when the welfare weights of "the poor" are higher than the welfare weights of "the rich." Second, even if the two conditions are not satisfied, it is conceivable that an empirical application of the sufficient statistics test reveals negligible gains from a reform towards a more sector-specific climate policy. Whether or not this is the case turns the question of this paper into a pragmatic one: are magnitudes such that a deviation from a uniform Carbon price is really paying off? This is then no longer a question on the principles of climate policy. While these principles are the focus the paper, the sufficient statistics formulas that it derives could be used in an empirical application to provide an answer also to the pragmatic question.

Equity considerations. We also evaluate deviations from a uniform price on Carbon with welfare weights that are decreasing in disposable income. We first proceed under the assumption of *proportional fiscal externalities*, so that, with distributive indifference, there would no be no reason to deviate from a uniform price. With a distributive objective, by contrast, it is desirable to move away from a uniform Carbon tax so that there are higher taxes on a good that is consumed mostly by "the rich" (irrespectively of whether that is the green or the brown good) and subsidies on a good consumed mostly by "the poor". We show, moreover, that this conclusion can also be obtained when fiscal externalities are not proportional. More specifically, when the welfare gains from a reform of taxes on the unspecific good are small, then, again, it is desirable to move towards higher taxes on goods consumed mostly by "the rich" and towards subsidies for goods consumed mostly "the poor." These results are obtained from a simplified version of the general setup that facilitates a comparison to the Ramsey model of taxation in it most

[3]The term fiscal externalities has been coined to describe the impact that the use of one tax instrument has on the revenue that is generated with some other instrument.

elementary form: There are constant returns to scale technologies and, as a consequence, taxes are fully borne by consumers. In the basic Ramsey model, the question is which sector should contribute how much to meeting the government's revenue target. Here, by contrast, the question is which sector should contribute how much to meeting the government's emission target. The above results imply that – relative to the benchmark of reaching emission targets with a uniform Carbon tax – sectors that produce goods mostly consumed by "the poor" should contribute less to meeting the government's emission target, whereas sectors that produce goods mostly consumed by "the rich" should contribute more.

Outline. The next section discusses related strands of the literature. The formal model is introduced in Section 3. The main results of the paper can be found in Section 4. The last section contains concluding remarks. Formal proofs are in part D of the Appendix.

2 Related literature

This paper combines ideas from different strands of the literatures. The literature on the regulation of externalities often times uses a partial equilibrium model. The seminal paper by Weitzman (1974) is a prominent example. Firms differ in their marginal costs of abatement. With a tax on emissions or prices for tradable emission permits, firms will expand abatement up to the point where the marginal costs are equal to the tax or the price of an emission permit. If the tax or the price is, moreover, equal to the marginal social benefit of abatement, a first-best outcome results. In particular, marginal costs of abatement are equalized across firms, with the implication that the economy-wide costs of abatement are minimized. In this framework, the case for a uniform price on Carbon is compelling. Treating firms in different sectors differently can only make things worse.

In the partial equilibrium model, abatement is the only economic activity. This paper enriches this framework. Firms are producing consumption goods and emissions are a by-product. The firms' incentives to avoid emissions therefore depend not only on how high green taxes or prices for emission permits are. They also depend on the demand for their final product. In the model introduced below, an increase in demand goes together with an increased effort to avoid emissions. Therefore also the commodity tax system – which affects the incentives of consumers to buy one good or another – matters for climate policy. The comparative statics properties of this framework are broadly consistent with the patterns documented in Känzig (2023). At the firm level, employment, output and emissions are all decreasing when carbon prices go up.

General equilibrium effects are a topic of its own in the analysis of tax incidence. A seminal paper that focussed on corporate taxation is Harberger (1962). More recently, Sachs et al. (2020) focussed on general equilibrium effects in the labour market. Their analysis uses a primal approach, i.e. the analysis focusses on allocations that respect incentive and resource constraints. This paper uses the dual approach and, hence, there is a need to run the analysis through market-clearing conditions that pin down an equilibrium price vector that depends on the specification of climate policy. The benefit is that we obtain an explicit characterization of how producer and consumer prices are affected by a change of climate policy and also of how the surplus that is generated in the various markets of the economy is affected. This gives a general equilibrium analogue to Harberger's famous measure of the economic surplus that is lost when distortionary taxes are used, Harberger's triangle.

Bovenberg and Goulder (1996) present a second best analysis of optimal environmental taxes.[4] Their setting also gives rise to general equilibrium effects. An important difference to the analysis in this paper is that Bovenberg and Goulder analyze a model with a representative household. The same is true for the analyses of optimal dynamic Carbon taxes by Golosov et al. (2014b) and Barrage (2019). Thus, these frameworks are not suited to studying the distributive implications of climate policy. Douenne et al. (2024) study climate policy in dynamic quantitative model with heterogenous households and with an explicit focus on its distributive implications. In their framework the assumptions underlying the famous Atkinson-Stiglitz result, discussed in more detail below, are satisfied so that there is no rationale for a deviation from a uniform price of Carbon.

Distributive issues are also important in the analysis of optimal environmental taxes by Cremer et al. (1998), Jacobs and de Mooij (2015), Jacobs and van der Ploeg (2019) and, more recently, Pai and Strack (2022) and Ahlvik et al. (2024). Their settings, however, do not give rise to general equilibrium effects. Another difference to the analysis in this paper is that, here, there is no attempt to characterize an optimal externality and an optimal Pigouvian correction. Instead, there is an exogenous emission target of the government. This is motivated by the Paris climate agreement that led to *Nationally determined contributions (NDAs).* We ask what such national emission targets, if they were really binding, would imply for the design of climate policy. Neither do we proceed under the assumption that those NDAs are optimally set, nor do we exclude this possibility.

Optimal sector specific taxes are characterized in Ramsey models of taxation, albeit with the objective to generate a predetermined level of tax revenue. Here, by contrast, the focus is on reaching a target level of emissions. As is well known, the Ramsey approach has been criticized by Atkinson and Stiglitz (1976). When consumption preferences are such that all individuals would spend a given amount of disposable income in the same way, then any tax system that involves differential commodity taxes is dominated by one

[4]In the presence of pre-existing distortions, it may desirable that policy choices deviate from first-best principles, see Lipsey and Lancaster (1956) for a seminal reference. It has been observed that this logic can also justify deviations from a uniform price of Carbon that would be desirable in the absence of such frictions, see e.g. Hoel (1996) for an analysis in which deviations from a uniform price of Carbon are desirable when trade-policy is restricted, but not when it is flexible. A related result is proven in this paper: With distributive indifference, a deviation from uniform carbon pricing is desirable only if there is a pre-determined non-linear income tax.

that relies only on income taxation.[5] Hellwig and Werquin (2024) trace out some of the empirical implications that are implied by the assumptions of Atkinson-Stiglitz and show that they are inconsistent with actual consumption data. Ferey et al. (2024) present optimal tax formulas for non-linear income and consumption taxes that apply when the Atkinson-Stiglitz assumptions are not satisfied. This paper follows Saez (2002) in the modelling of consumption spending: Utility is separable between homothetic consumption utility on the one hand and effort costs on the other. Moreover, the consumption utility part may be different for different individuals. While this nests the Atkinson-Stiglitz specification as a special case, it allows for heterogeneity in consumption preferences; for instance, it allows for the possibility that incomes and preferences for green rather than brown consumption goods are positively correlated.[6]

While this paper uses arguments and techniques from the literature on optimal taxation, there are still notable differences to the workhorse models in this literature. Here, the government is constraint by the need to reach an emission target. Also, firms make profits and make an effort to reduce emissions. These choices depend on the properties of the tax system. Firm profits are a source of income for some, but not for all households. Thus, there is inequality in incomes beyond the inequality in labour incomes. A difference between this paper and much of the related literature in public finance is, moreover, that there is no attempt to characterize an optimal tax system. Instead, the question is whether a particular benchmark, a uniform price on Carbon, is desirable.

Much the literature on optimal commodity taxes uses a mechanism design approach. This makes it possible to characterize optimal tax systems without ad hoc assumptions on the functional form of an optimal tax schedule. It detaches the normative theory, however, from its positive counterpart. There is then no explicit characterization of how market prices and quantities respond to changes of the tax system. This paper uses the dual approach and therefore contains such a characterization. It then looks at perturbations of climate policy starting from competitive equilibrium allocations. Perturbation arguments are frequently used in the analysis of non-linear income tax systems, see Piketty (1997), Saez (2001), Golosov et al. (2014a) for important references. A contribution of this paper is to bring this approach to the analysis of climate policy.

[5]See Laroque (2005) for a simple proof and Doligalski et al. (2023) for a generalization of the argument that covers both the production efficiency theorem of Diamond and Mirrlees (1971) and the result of Atkinson and Stiglitz (1976) on the non-desirability of differential commodity taxation. For related results in the setting of this paper, see part A of the Appendix.

[6]It implies, however, that Engel curves are linear. For an analysis of optimal policy with non-linear Engel curves, see Jacobs and van der Ploeg (2019).

3 The model

3.1 Households

Preferences. There is a unit mass of individuals or households. Individuals have preferences that are represented by a utility function

$$u(x_c, \chi(\beta x_g, x_b)) - k(y_l, \omega) \ .$$

The consumption utility u depends on two arguments, the quantity consumed of the unspecific consumption good x_c, and a subutility χ which results from the combination of green and brown consumption goods, x_g and x_b. The brown and the green good are assumed to be imperfect substitutes. As an example, think of x_g as "kilometers travelled by train", of x_b as "kilometers travelled by plane" and of χ as the subutility from travelling. The function χ depends on a parameter β so that the willingness to pay for the green good is increasing in β. We assume that possible values of β belong to an interval $[\underline{\beta}, \bar{\beta}]$ with $\underline{\beta} > 0$. The functions u and χ are both assumed to be homothetic. We denote by y_l an individual's labour supply, k is an effort cost function, and ω is a measure of productive abilities that affects the marginal effort costs. We assume that the cross-derivative k_{12} is negative so that higher ω-types have lower marginal effort costs. Otherwise, k is assumed to satisfy the usual Inada conditions.

Budget constraint. We denote the vector of consumer prices by $q = (q_c, q_g, q_b)$. The corresponding producer prices are denoted by p_c ,p_g and p_b. Commodity taxes drive a wedge between consumer and producer prices so that

$$q_c = (1 + t_c)\, p_c \quad q_g = (1 + t_g)\, p_g \quad \text{and} \quad q_b = (1 + t_b)\, p_b \ .$$

We are, inter alia, interested in the desirability of differential commodity taxation. Thus, we consider the possibility to tax green and brown consumption at rates that are different from t_c. When individuals supply y_l units of labour, they realize a gross labour income of $p_w\, y_l$, where p_w is the wage rate. Labour income is taxed according to a non-linear income tax schedule: $T_l : p_w\, y_l \mapsto T_l(p_w\, y_l)$. The tax schedule T_l is assumed to be twice continuously differentiable. Possibly, individuals also realize "capital income" from the shares they hold in the economy's firms. We write $s = (s_c, s_g, s_b)$ for a generic portfolio and $\Pi = (\Pi_c, \Pi_g, \Pi_b)'$ for the column vector that list the profits realized in the different sectors in the economy. A generic "capital income" can then be written as the scalar product $s\ \Pi$. The government redistributes net tax revenues $\mathcal{R}$ in a lump-sum fashion. Possibly, this revenue is generated by taxes on Carbon emissions, discussed in more detail below. Taking all this into account an individual's budget constraint reads as

$$q_c\, x_c + q_g\, x_g + q_b\, x_b \quad \leq \quad p_w\, y_l - T_l(p_w\, y_l) + s\ \Pi^E + \mathcal{R}^E \ . \tag{1}$$

When individuals choose labour supply and consumption demand they hold expectations about the profits and the tax revenue that will contribute to their disposable income, as indicated by the superscript E. When we formally state the definition of an equilibrium below, we will add the requirement that these expectations are correct.

Utility maximization. Individuals choose $x = (x_c, x_g, x_b)$ and y to maximize utility subject to the budget constraint in (1). It will prove useful to decompose this problem into an inner and an outer problem. The inner problem is to maximize $u(x_c, \chi(\beta x_g, x_b))$ for a given level of disposable income c. Hence, the budget constraint for the inner problem is

$$q_c\, x_c + q_g\, x_g + q_b\, x_b \quad \leq \quad c\ .$$

The solution $x^* = (x_c^*, x_g^*, x_b^*)$ to this problem depends on the prices of consumption goods $q = (q_c, q_g, q_b)$, the preference parameter β, and the disposable income c. The indirect utility function v is defined by

$$v(c, \beta, q) = u(x_c^*(c, \beta, q), \chi(\beta x_g^*(c, \beta, q), x_b^*(c, \beta, q)))\ .$$

The outer problem is to choose c and y_l to maximize

$$v(c, \beta, q) - k(y_l, \omega) \quad \text{s.t.} \quad c = p_w\, y_l - T_l(p_w\, y_l) + s\, \Pi^E + \mathcal{R}^E\ .$$

Let $\theta = (\beta, \omega, s)$ be a shorthand for an individual's type. The solution to the outer problem can be written as $c^*(\theta, \Pi^E, \mathcal{R}^E, q, p_w, T_l)$ and $y_l^*(\theta, \Pi^E, \mathcal{R}^E, q, p_w, T_l)$. Individual demand for the various consumption goods is obtained by inserting $c^*(\theta, \Pi^E, \mathcal{R}^E, q, p_w, T_l)$ for c in $x_c^*(\beta, c, q)$, $x_g^*(\beta, c, q)$ and $x_b^*(\beta, c, q)$.

Comparative statics of individual choices. With u homothetic the inner problem can be written as: Choose $z_c = \frac{x_c}{c}$, $z_g = \frac{x_g}{c}$ and $z_b = \frac{x_b}{c}$ to maximize $c\; u(z_c, \chi(z_g, z_b))$ subject to $q_c\, z_n + q_g\, z_g + q_b\, z_b \leq 1$. The optimal choices of $z = (z_n, z_g, z_b)$ then depend only on the consumer prices q and the preference parameter β. Indirect utility is therefore given by

$$v(c, \beta, q) = c\; u(z_c^*(\beta, q), \chi(z_g^*(\beta, q), z_b^*(\beta, q))) =: c\; \tilde{v}(\beta, q)\ . \tag{2}$$

We henceforth refer to $\tilde{v}(\beta, q)$ as the marginal utility of disposable income.

Saez (2002) has also invoked a homothetic consumption utility function combined with the assumption that this utility function may be different for different individuals. Here, this corresponds to the heterogeneity in β. Saez did not impose further assumptions on the outer utility function that governs the tradeoff between consumption utility and the costs of productive effort. Here, we assume that this overall utility function is additively separable. Hence, the outer problem can be written as

$$\max_{y_l} \quad (p_w\, y_l - T_l(p_w\, y_l) + e)\tilde{v}(\beta, q) - k(y_l, \omega)\ , \quad \text{where} \quad e := s\, \Pi^E + \mathcal{R}^E\ .$$

This choice brings a similarity to the analysis of the Mirrelesian income tax problem in Diamond (1998). Diamond assumed quasi-linear in consumption preferences to the effect of removing income effects from the analysis of the optimal income tax problem. Here, the individual's capital incomes and the transfers that individuals receive, as summarized in e, do not affect the solution to the outer problem. The solution to the outer problem is affected only by the marginal utility of disposable income, which is non-decreasing in β, and the marginal effort costs, which are decreasing in ω. This gives rise to monotone comparative statics, see Milgrom and Shannon (1994). Labour incomes are therefore increasing in β and ω. These observations are summarized in the following Lemma that we state without proof.

Lemma 1 *Suppose that $p_w\, y_l - T_l(p_w\, y_l)$ is a non-decreasing function of y_l.*

i) The marginal utility of disposable income is constant: $v_c(c, \beta, q) = \tilde{v}(\beta, q)$ and $v_{cc}(c, \beta, q) = 0$, where v_c and v_{cc} denote, respectively, the first and the second derivative of the indirect utility function with respect to the level of disposable income.

ii) The utility-maximizing labour supply y_l^* does not depend on s_c, s_g, s_b, Π^E and $\mathcal{R}^E$.

iii) The marginal utility of disposable income is increasing in β and decreasing in the consumer prices q_c, q_g, and q_b.

iv) The utility-maximizing levels of disposable income c^* and labour supply y_l^* are non-decreasing functions of β and ω and non-increasing functions of q_c, q_g, and q_b.

Thus, the "capital income" that individuals realize and the tax revenues that the government might redistribute are without consequence for individual labour supply. Still there are income effects. If an individual's disposable income goes up, the consumption of all goods scales up while the composition of the consumption basket remains the same as individuals get poorer or richer. There is, however, heterogeneity in the composition of the basket due to heterogeneity in the preference for green goods, parameterized by β. Higher values of β, moreover, imply a higher marginal utility of disposable income. This increases earnings incentives. Thus, ceteris paribus, individuals with a higher taste for green goods do not earn less than people with a higher taste for brown goods. By the same logic, higher consumer prices lower the marginal utility of disposable income and thus reduce earnings incentives. Finally, as in Mirrleesian models of income taxation, earnings incentives increase in productive abilities, so that both disposable income and labour earnings are non-decreasing functions of ω. An implication of the Lemma is that, ceteris paribus, the (hypothetical) redistribution of one dollar from a low β person to a high β person would reduce the demand for the brown good and increase the demand for the green good: The marginal carbon footprint, i.e. the footprint per unit of disposable income, of a high β-type is lower than the marginal carbon footprint of a low β-type. At the same time, it would involve redistribution from a low income person to a high income person.

Aggregate labour supply, Aggregate consumption demand. Let Φ_θ be the *cdf* that describes the joint distribution of productive abilities ω, preferences for green consumption β and "capital incomes" s. Aggregate labour supply can then be written as

$$Y_l(\Pi^E, \mathcal{R}^E, q, p_w, T_l) = \mathbf{E}_\theta \left[y_l^*(\theta, \Pi^E, \mathcal{R}^E, q, p_w, T_l) \right] \ ,$$

where the operator $\mathbf{E}_\theta$ indicates the computation of an expectation using the distribution Φ_θ. We define the aggregate demand for the different consumption goods analogously. Henceforth $X_c(\Pi^E, \mathcal{R}^E, q, p_w, T_l)$, $X_g(\Pi^E, \mathcal{R}^E, q, p_w, T_l)$ and $X_b(\Pi^E, \mathcal{R}^E, q, p_w, T_l)$ denote, respectively, aggregate demand for the unspecific consumption good, the green and the brown good.

3.2 Firms

There are three sectors in the economy, indexed by $j \in \{c, g, b\}$, where c stands for the sector producing the unspecific consumption good, g stands for the green sector and b for the brown sector. The firms in any one sector j produce the sector's final output good, using labour l as the only input. In addition, they can invest resources r to reduce the emission intensity of their production. Firms differ in the cost of the investment that is needed to reduce the emission intensity of their production.[7]

Profit-maximization. The profit-maximization problem of a generic firm in sector j is to choose l and r to maximize

$$p_j \ \alpha \ f_j(l) - p_w \ l - t_{je}\Big(e_{j0} - a_j(r)\Big) \ \alpha f_j(l) - p_c \ \gamma \ r \ , \quad \text{with} \quad a_j(0) = 0.$$

We now explain the various terms that enter this expression. A firm in sector j, sells goods at a price p_j to the market. The production function f_j is assumed to satisfy the usual Inada conditions. We will sometimes refer to is an iso-elastic production function

$$f_j(l) = \frac{1}{1 - \frac{1}{\sigma_j}} l^{1 - \frac{1}{\sigma_j}}$$

where σ_j is the elasticity of substitution for firms in sector j. Firms in a given sector j are assumed to differ in their factor productivity α. The wage bill of a firm that hires l units of labour is $p_w l$.

Carbon emissions are a byproduct of production. The parameter e_{j0} gives the emission intensity of a firm in sector j if it does nothing – as captured by the subscript 0 – to avoid emissions. The possibility of emission avoidance is captured by the abatement function a_j. It is non-negative, increasing in r and concave. It is bounded from above by e_{j0} and

[7] The heterogeneity of firms in terms of their abatement costs is often invoked in justifications of a uniform price on Carbon. The logic is that a uniform price is efficient in that it gives firms with low abatement costs incentives to cut emissions, whereas firms with high abatement costs pay the price of carbon.

satisfies the Inada conditions. The more resources r are devoted to emission avoidance, the lower are the emissions $e_{j0} - a_j(r)$ per unit of output. Emissions in sector j are taxed at rate t_{je}. From the firm's perspective, the tax rate t_{je} can equivalently be interpreted as a price for an emission permit. In this interpretation, a firm combines two factors of production, labour and emission permits, in the production of its final output good. Thus, in the given setting, the classical question on the desirability of *production efficiency*[8] can be posed as the question whether an optimal policy should distort the relative prices of labour inputs and emission permits away from some first-best benchmark.[9]

The parameter γ is a measure of how many resources a firm needs to invest to achieve a given level of emissions reduction. Firms with high γ have a high cost of emission avoidance. We treat the unspecific consumption good as a multi-purpose good that can be used both for consumption and for investments into emission avoidance. Thus, a firm that wants to reduce its emission intensity by $a_j(r)$ needs to spend $\gamma\, r$.

A firm's decision how much labour to hire and hence how much to produce and its decision how much to invest into emission avoidance are interdependent. Consider the first order conditions that characterize the profit maximizing choices l^* and r^*. The first order condition for the choice of labour inputs is

$$\Big(p_j \; - t_{je}(e_{j0} - a_j(r^*))\Big) \, \alpha \, f_j'(\, l^*) \quad = \quad p_w \;, \tag{3}$$

and the first order condition for emission avoidance is

$$\frac{p_c \, \gamma}{a_j'(r^*) \, \alpha \, f_j(l^*)} \quad = \quad t_{je} \;. \tag{4}$$

These are two equations in two unknowns. In the absence of emissions taxes, the first order condition in (3) is the familiar condition that the value of the marginal product of labour is equal to the wage rate. With an emissions tax, the value p_j is reduced by the emission costs that come with an expansion of employment and production. These costs are lower the more the firm invests into emission avoidance. Thus, a higher level of r^* goes together with a higher level of l^*. The first order condition in (4) also gives rise to a complementarity between output and employment on the one hand and emission avoidance on the other. It has, on the left hand side, the marginal cost of avoiding one unit of emissions and, on the right hand side, the price of an emission permit, or equivalently, the taxes that can be saved when one unit of emissions is avoided. An inspection of (4) shows that a higher level of l^* reduces the marginal cost of avoidance. Hence, firms who opt for a larger scale of production also devote more resources to the avoidance of emissions.

Comparative statics of firm behavior: Output, Employment, Investment and Emissions. To understand how form behavior changes when taxes and prices change

[8]See the production efficiency theorem by Diamond and Mirrlees (1971) and, for a more recent treatment, Doligalski et al. (2023).

[9]In Part A of the Appendix, we clarify what the appropriate first-best benchmark is in this case.

it is useful to decompose the firm's profit-maximization problem into an inner and an outer problem. For the inner problem, the employment level l is taken as given and the firm chooses r to maximize

$$-t_{je}(e_{j0} - a_j(r))\,\alpha f_j(l) - p_c\,\gamma\,r$$

The solution to this problem is denoted by $r^*(l, p_c, t_{je}, \gamma)$. It is straightforward to verify that r^* is increasing in l and t_{je} and decreasing in p_c and γ. The outer problem then is to choose l to maximize

$$p_j\;\alpha\;f_j(l) - \;p_w l - t_{je}\,(e_{j0} - a_j(r^*(l,\cdot)))\,\alpha f_j(l)) - p_c\;\gamma\;r^*(l,\cdot)\;.$$

We denote the solution to this problem by $l^*(p_j, p_w, p_c, t_{je}, \gamma, \alpha)$. It is straightforward to verify that l^* is increasing in p_j and α and decreasing in p_w. The complementarity of the investment and the labour choice implies, moreover, that l^* is decreasing in p_c and γ, as r^* is decreasing in these variables.

Lemma 2 *Let $f_j(l) = \frac{1}{1-\frac{1}{\sigma_j}} l^{1-\frac{1}{\sigma_j}}$. Denote profit-maximizing emissions by*

$$\mathbf{e}_j^*(p_j, p_w, p_c, t_{je}, \gamma, \alpha) := \Big(e_{j0} - a(r^*(l^*(\cdot), \cdot))\Big)\;\alpha\;f_j(l^*(\cdot))\;.$$

The function $\mathbf{e}_j^*$ has the same comparative statics properties as the function l^*: It is increasing in p_j and α and decreasing in p_w and t_{je}. It is decreasing in p_c and γ.

Thus, essentially, at the level of an individual firm, the comparative statics of emissions, output, employment and investment all have the same sign. Anything that makes the firm expand output, employment and investment also implies more emissions. Anything that makes the firm reduce emissions goes together with a down-scaling of all its economic activities.

These comparative statics results can be related to discussions on whether the "green transformation" of the economy – the change of technologies so that production processes become cleaner – can be a source of economic growth. Through the lens of the model, the answer is "no" if the comparison is to a benchmark economy that has emissions which are too high. A change of policy that brings down emissions will then also bring down output, employment and investments into greener technologies. The answer is "yes" if the comparison is to a benchmark economy that has to cut emissions while operating with fixed technologies. The possibility to invest then implies that output and employment are higher than they would otherwise be.

What distinguishes the green sector from the brown sector? So far the sector names "unspecific", "green" and "brown" have been labels with no meaning. There are different conceivable ways to distinguish sectors according to how dirty they are. For instance, if for any given r,

$$e_{b0} - a_b(r) \quad > \quad e_{c0} - a_c(r) \quad > \quad e_{g0} - a_g(r)\;,$$

then, emissions per unit of output are largest in the brown sector and smallest in the green sector. An alternative is to order them according to their marginal cost of avoidance. If for any given α, γ, l and r,

$$\frac{\gamma}{a_b'(r)\ \alpha\ f_b(l)} \quad > \quad \frac{\gamma}{a_c'(r)\ \alpha\ f_c(l)} \quad > \quad \frac{\gamma}{a_g'(r)\ \alpha\ f_g(l)} \ ,$$

then the marginal avoidance costs are lowest in the green sector and highest in the brown sector. The analysis in this paper does not presume that it is possible to order sectors in this way, but it is consistent with such a possibility. The only assumption that will be used in the formal analysis below is that

$$e_{b0} \quad > \quad e_{c0} \quad > \quad e_{g0} \ .$$

Aggregation. For later use in the analysis of competitive equilibria, we define labour demand, goods supply and the demand for emission permits both at the sector and the aggregate level. We start with individual firm behavior. The choices of a firm in sector j depend on its characteristics $\theta_j = (\alpha, \gamma)$, the prices $p^j = (p_j, p_w, p_c)$ it is facing and the sector specific tax t_{je}. We denote, respectively, by $l^*(\theta_j, p_j, p_w, p_c, t_{je})$ and $r^*(\theta_j, p_j, p_w, p_c, t_{je})$ the firm's labour demand and the resources that it invests to avoid emissions. The firm's supply of good j is then given by

$$y_j^*(\theta_j, p^j, t_{je}) = \alpha f_j(l^*(\theta_j, p^j, t_{je}))$$

and its emissions are equal to

$$\mathbf{e}_j^*(\theta_j, p^j, t_{je}) = \Big(e_{j0} - a_j(r^*(\theta_j, p^j, t_{je}))\Big)\ y_j^*(\theta_j, p^j, t_{je}) \ ,$$

where

$$\gamma\, r^*(\theta_j, p^j, t_{je})$$

is the firm's abatement effort, measured in expenditures for the unspecific consumption good. Profits are then given by

$$\begin{aligned} \pi_j(\theta_j, p^j, t_{je}) \quad &= \quad p_j\ y_j^*(\theta_j, p^j, t_{je}) - p_w\ l^*(\theta_j, p^j, t_{je}) \\ &\quad -p_c\ \gamma\ r^*(\theta_j, p^j, t_{je}) - t_{je}\ \mathbf{e}^*(\theta_j, p^j, t_{je}) \ . \end{aligned}$$

Labour demand, goods supply, emissions and profits at the sector level. Let Φ_j be the *cdf* that represents the distribution of firm characteristics in sector j. Total labour demand by firms in sector j is denoted by

$$L_j(p^j, t_{je}) = \mathbf{E}_j\left[l^*(\theta_j, p^j, t_{je})\right] \ .$$

Analogously we define by $Y_j(p^j, p_c, t_{je})$ the sector's goods supply, by $\mathcal{E}_j(p^j, p_c, t_{je})$ the sector's demand for emission permits and by $R_j(p^j, p_c, t_{je})$ the sector's demand for the unspecific consumption good. Profits in sector j are denoted by $\Pi_j(p^j, p_c, t_{je})$.

Aggregate labour demand and the aggregate demand for emission permits. Let $p = (p_c, p_g, p_b, p_w)$ be the economy's producer price system. Let $p^g = (p_g, p_c, p_w)$ and $p^b = (p_b, p_c, p_w)$ be the set of prices that are relevant for firms in the green and the brown sector. Analogously, we define $p^c = (p_c, p_w)$. Aggregate labour demand can then be written as

$$L(p, t_e) = L_c(p^c, t_{ce}) + L_g(p^g, t_{ge}) + L_b(p^b, t_{be})$$

where $t_e = (t_{ce}, t_{be}, t_{ge})$ is the collection of sector-specific emissions taxes. Analogously, we denote the overall demand for emission permits by $\mathcal{E}(p, t_e)$, the resources devoted to the greening of technologies by $R(p, t_e)$ and by

$$\Pi(p, t_e) = \Big(\Pi_c(p^c, t_{ce}), \Pi_g(p^g, t_{ge}), \Pi_b(p^b, t_{be})\Big)$$

the vector of sectoral profits.

3.3 Competitive equilibrium given climate policy

Overall tax policy consists of collection of taxes that appear in the individuals' budget constraints (t_c, t_b, t_g, T_l) and the emission taxes $t_e = (t_{ce}, t_{be}, t_{ge})$ that affect the choices of firms. Differential commodity taxation is reflected in the possibility to tax green and brown consumption goods at rates that differ from t_c. Sector-specific taxation is captured by the possibility to tax Carbon emissions at sector-specific rates. We use $\mathcal{T} = (t_c, t_b, t_g, T_l, t_e)$ as a shorthand for all the policy instruments. We assume that there is an overall emission target $\bar{\mathcal{E}}$ so that

$$\mathcal{E}(p, t_e) \leq \bar{\mathcal{E}} \ . \tag{5}$$

and that the government considers only policies which reach this target.[10] Tax revenue, of any, is rebated lump sum. Given a tax policy, a price system, and expectations about tax revenues and profits Π^E and $\mathcal{R}^E$, aggregate tax revenue $\mathcal{R}(p, \mathcal{T})$ is given by

$$\begin{aligned} \mathcal{R}(p, \mathcal{T}) \ &= \ \textstyle\sum_{j \in \{c,b,g\}} t_j \ p_j \ X_j(\Pi^E, \mathcal{R}^E, q, p_w, T_l) \\ &\quad + \ \textstyle\sum_{j \in \{c,b,g\}} \ t_{je} \ \mathcal{E}_j(p^j, t_{je}) \\ &\quad + \ \mathbf{E}_\theta \Big[T_l(p_w \ y_l^*(\theta, \Pi^E, \mathcal{R}^E, q, p_w, T_l)) \Big] \ . \end{aligned} \tag{6}$$

By the following Lemma, there is one and only one level of expected tax revenue $\mathcal{R}^E$ that is "correct", i.e. consistent with the way in which actual tax revenue depends on expected tax revenue. The proof follows from an application of Brouwer's fixed point theorem.

[10] In political practice, it is disputable whether emission targets – such as those associated with the Paris climate protocol – are really binding.

Lemma 3 *Suppose that there is an upper bound $\bar{\mathcal{R}}$ on the tax revenue that can be collected. Suppose that all consumption goods are normal goods. Then, for given prices p, tax policy $\mathcal{T}$ and expected profits Π^E, there is one and only level of tax revenue so that $\mathcal{R}(p, \mathcal{T}) = \mathcal{R}^E$.*

Equilibrium. Given a tax policy $\mathcal{T}$, a price system $p = (p_c, p_g, p_b, p_w)$ is an equilibrium price system if the following conditions are met: The labour market clears, $L(p, t_e) = Y_l(\Pi^E, \mathcal{R}^E, q, p_w, T_l)$, the market for the unspecific consumption good clears $X_c(\Pi^E, \mathcal{R}^E, q, p_w, T_l) + R(p, t_e) = Y_c(p^c, t_{ce})$, the markets for the brown and the green good clear, $X_j(\Pi^E, \mathcal{R}^E, q, p_w, T_l) = Y_j(p^j, t_{je})$, for all $j \in \{b, g\}$, and expectations are correct $\mathcal{R}^E = \mathcal{R}(p, \mathcal{T})$ and $\Pi^E = \Pi(p, t_e)$ where $R(p, \mathcal{T})$ is defined in equation (6) and $\Pi(p, t_e)$ is the vector that lists aggregate profits in the different sectors of the economy.

Lemma 4 *If the goods markets clear and expectations are correct, then the labour market clears.*

The lemma, which is simply a version of Walras' law for the given economy, simplifies the equilibrium characterization. If all goods markets clear and expectations are correct, the labour market clears too, so that the equilibrium characterization can focus on the goods market clearing conditions.

3.4 Existence, uniqueness and tax incidence: An example

For the remainder of this section and without further mention we impose the assumptions that the consumption utility function u is Cobb-Douglas, i.e.

$$u(x_c, \chi(\beta x_g, x_b)) = x_c^{1-\nu} \chi(\beta x_g, x_b)^{\nu} \ , \tag{7}$$

and that the subutility χ gives rise to a constant elasticity of substitution,

$$\chi(\beta x_g, x_b) = \left(\beta x_g^{1-\varepsilon_\chi} + x_b^{1-\varepsilon_\chi}\right)^{\frac{1}{1-\varepsilon_\chi}} \ . \tag{8}$$

We use this example to demonstrate that the models primitive's can be specified in such a way that competitive equilibria exist and are unique. Once this is established we provide comparative statics results on how equilibrium prices respond when consumption or emission taxes change.[11]

[11] A more general proof of existence and uniqueness does not seem to be available. By the Sonnenschein-Mantel-Debreu theorem, excess demand functions have little structure in general. For an exchange economy with two goods and CES consumer preferences a proof of existence and uniqueness can be found in Mas-Colell et al. (1995). Proposition 1 extends this result in various ways: There is production, and there are profits in equilibrium, there are three rather than two consumption goods, there are linear taxes on both inputs and outputs and there is a non-linear income tax.

The above assumptions are not used to establish the main results in the in the subsequent section. Moreover, there are ways to obtain comparative statics results on equilibrium prices also when there are multiple equilibria.[12] Thus, the example imposes more structure than is necessary. It serves an illustrative purpose. The results in the subsequent section build on the changes of equilibrium prices in response to changes in the specification of climate policy. The example is useful to get an understanding of what these changes of equilibrium prices can look like.

Proposition 1 *There exists $\bar{\nu}$ so that for $\nu < \bar{\nu}$, for any vector of tax rates, there is a unique equilibrium price vector.*

The proof proceeds as follows: We take the wage rate as the numeraire, so that only the prices p_c, p_g and p_b need to be determined in equilibrium. We then fix p_b and p_c at arbitrary levels and show that there is unique price p_g that clears the market for the green good. As we vary p_b, this partial equilibrium value of p_g adjusts. So, there is a possibility to vary both p_b and p_g while keeping the market for the green good in partial equilibrium. We further observe that any variation that involves a higher/ lower level of p_b lowers/ increases excess demand in the market for the brown good. Thus, we can bring the market for the brown good into partial equilibrium, while maintaining the partial equilibrium in the market for the green good.[13] All this holds for arbitrary values of p_c. As a final step we bring p_c to the level that clears the market for the unspecific consumption good while adjusting p_g and p_b so that both the market for the green good and the market for the brown good both remain in partial equilibrium. Ultimately we have found a general equilibrium price vector in this way. The monotonicity of excess demand functions in their "own" price implies, moreover, that this general equilibrium price vector is unique.

Tax incidence: Comparative statics of equilibrium prices. The assumption used to establish existence and uniqueness of equilibrium also imply that predictions on whether equilibrium prices rise or fall in response to changes of taxes can be obtained analytically. The problem in general is that there are three excess demand functions and that the whole vector of equilibrium prices depends on all the policy variables collected in $\mathcal{T}$. Under the assumptions for Proposition 1, however, one can get at these the predictions by (i) treating p_g and p_b as fixed parameters in the excess demand function for the unspecific consumption good and (ii) be treating disposable incomes and p_c as fixed in the excess demand functions for the green and the brown good.

[12] See e.g. Bierbrauer (2014) for an analysis of tax incidence on financial markets in the presence of multiple equilibria.

[13] As shown in the proof of Proposition 1, for this to be possible the excess demand functions for the green and the brown good need to satisfy a single crossing property. This single property is then shown to hold for ν sufficiently small, so that labour market outcomes, disposable incomes and earnings do not depend too much on the prices of the green and the brown good.

Proposition 2

1. *The equilibrium value of p_c is decreasing in t_c and increasing in t_{ce}. The equilibrium value of q_c is increasing in t_c and t_{ce}.*

2. *The equilibrium values of p_g, q_g, p_b and q_g are increasing in t_{ge} and t_{be}.*

3. *The equilibrium value of p_b is decreasing in t_b and the equilibrium value of q_b is increasing in t_b. The equilibrium values of p_g and q_g are increasing in t_b.*

4. *The equilibrium value of p_g is decreasing in t_g and the equilibrium value of q_g is increasing in t_g. The equilibrium values of p_b and q_b are increasing in t_b.*

If the government taxes emissions in the large sector more heavily (t_{ce} goes up), the unspecific consumption good becomes more expensive for consumers and their marginal utility of income goes down. Consequently, there is also a contraction of labour supply. The same happens when the government taxes the sector's output more heavily (t_c goes up.) There is a difference, however, in how producer prices react. When t_{ce} goes up, production costs increase and there is less supply at given prices. To restore equilibrium, producer prices have to increase. When t_c goes up, there is less demand at given producer prices. To restore equilibrium, producer prices have to fall. In any case, the comparative statics results in previous sections on how individual consumers and firms respond to price changes imply that output, employment, investment and emissions go down in the large sector when either t_c or t_{ce} goes up.

If the government raises the tax on the use of fossil fuels in the airline industry (i.e. t_{be} goes up), but not in the railway industry (i.e. t_{ge} stays constant), then producer and consumer prices both in the brown and in the green sector go up. Prices for flight tickets increase because production has become more costly. This increases the demand for the green substitute so that also the prices of train tickets go up. Consequently, employment, investment and overall emissions go up in the railway-industry. The mirror image is that employment, investment and overall emissions go down in the airline-industry.

Proposition 2 does not cover all effects of tax changes. First, with ν small, what happens in the small markets for the brown and the green good doesn't matter much for the outcome of the large market for the unspecific good. The Proposition therefore does not include the effects that changes of t_g, t_b, t_{ge} or t_{be} have on p_c and q_c. They effects are close to zero anyway. Second, what happens in the large market has consequences in the small markets for the brown and the green good. However, there are various channels and overall effects are difficult to sign unless stronger assumptions are imposed. What matters is the relative strength of effects on demand and supply in the green and the brown sector.

Socially responsible consumers. Suppose that more and more individuals become socially responsible consumers and use the train rather than the plane for short-distance travel. How would this affect equilibrium prices? A conceivable formalization is as follows: For every individual, β is replaced by $h(\kappa)\beta$, where h is some increasing function and κ can be interpreted as an extra "kick" to the preferences for the green good. Equilibrium prices then depend both on public policy $\mathcal{T}$ and the size of the kick κ.[14] The following Proposition clarifies how equilibrium prices respond to changes of κ.

Proposition 3 *The equilibrium values of p_g and q_g are increasing in κ. The equilibrium values of p_b and q_b are decreasing in κ.*

Thus, if more people use the train, train rides become more expensive and airline tickets become cheaper. If squared with Lemma 2 this implies, moreover, that output in he railway sector goes up with the consequence that there is an increased effort to reduce the emission intensity of railway travel. By contrast, output in the airline industry goes down, hence also the sector's investments into abatement. Thus, a shift of preferences towards green alternatives pushes incentives for emission avoidance in the green and the brown sector in opposite directions: The green sector makes an effort to become even greener. The brown sector makes less of an effort and therefore becomes even browner.

With a binding target level emissions, does it bring any good if consumers switch to greener goods out of a sense of social responsibility? If they reduce their individual carbon footprints, this does not imply that emissions go down overall, it just implies that someone else will increase his or her carbon footprint.[15] While this is true, the analysis in this paper suggests that an increase of the demand for green goods is not without consequence. The switch to green goods increases output, employment and investment in the green sector. The same variables shrink in the brown sector. When the overall demand for emissions falls as a consequence, then also the prices of emission permits or, equivalently, corrective taxes can go down. This lowers the firms production costs, again with positive implications for output and employment.

4 The main results

We perform a test for the desirability of a uniform price of Carbon that proceeds as follows: We consider a competitive equilibrium with (i) uniform emissions taxes, (ii) uniform commodity taxes, (iii) an arbitrary income tax schedule which are specified such that the emission target is reached. We then consider deviations from uniform emission taxes and/ or uniform commodity taxes and evaluate them with alternative social welfare functions. The evaluation focusses on deviations that respect the emission target; i.e. we

[14] Alternative ways of shifting the distribution of preferences towards higher levels of β are conceivable. The approach taken here has an advantage of simplicity in that it allows for a unified approach in the proofs of Propositions 2 and 3.

[15] Herweg and Schmidt (2022) argue that this discourages socially responsible consumers from switching to green alternatives, see Kaufmann et al. (2023) for a related discussion.

will consider only deviations to policies $\mathcal{T}$ with the property that

$$\mathcal{E}(p^*(\mathcal{T}), t_e) = \bar{\mathcal{E}} \ , \tag{9}$$

where, here and henceforth, we suppress the dependence of endogenous variables on the wage rate p_w. By Walras's law (recall Lemma 4), we can set $p_w = 1$ without loss of generality.

Social welfare. Consider a competitive equilibrium with consumer prices q^*. Let $c(\theta)$ be the disposable income and $y_l(\theta)$ the labour supply of a type θ household in the corresponding competitive equilibrium allocation. Social welfare is then given by

$$\mathcal{W} = \mathbf{E}_\theta[g(\theta)\ U(\theta)] \quad \text{where} \quad U(\theta) = c^*(\theta)\ \tilde{v}(\beta, q) - k(y_l^*(\theta), \omega) \ .$$

The function $g : \theta \mapsto g(\theta)$ specifies welfare weights as a function of the individuals' types. For later use, we introduce some shorthands: For the social marginal utility of disposable income of type θ, given a competitive equilibrium with consumer prices q^*, we write

$$\mathbf{g}(\tilde{v}(\beta, q^*), \theta) := g(\theta)\ \tilde{v}(\beta, q^*) \ .$$

The population average of this quantity, sometimes also referred to as the marginal value of public funds, is denoted by $\bar{\mathbf{g}}(\cdot) := \mathbf{E}_\theta[\mathbf{g}(\tilde{v}(\cdot), \theta)]$. Finally, the social marginal utility of income for the recipients of "capital income" from sector j is $\bar{\mathbf{g}}_{\Pi^j} := \mathbf{E}_\theta[\mathbf{g}(\tilde{v}(\cdot), \theta)\ s_j]$.

The test. The test now proceeds as follows: let $\tau_1 \in \{t_c, t_g, t_b, t_{ce}, t_{ge}, t_{be}\}$ and $\tau_2 \in \{t_c, t_g, t_b, t_{ce}, t_{ge}, t_{be}\}$ be two different tax rates. A policy change that respects the emission target needs to satisfy

$$\mathcal{E}_{\tau_1} d\tau_1 \ + \ \mathcal{E}_{\tau_2} d\tau_2 = 0 \quad \text{or} \quad \frac{d\tau_2}{d\tau_1} \quad = \quad -\frac{\mathcal{E}_{\tau_1}}{\mathcal{E}_{\tau_2}} \ ,$$

where $\mathcal{E}_{\tau_1}$ and $\mathcal{E}_{\tau_2}$ are, respectively, total differentials. They give the marginal impact on total emissions when the levels of the tax instruments τ_1 and τ_2 slightly increases.[16] For the sake of the argument, let $d\tau_1 > 0$ and $d\tau_2 < 0$. The welfare-implications of such a policy change are positive if

$$\mathcal{W}_{\tau_1} d\tau_1 \ + \ \mathcal{W}_{\tau_2} d\tau_2 > 0$$

or, equivalently, if

$$\mathcal{W}_{\tau_1} \ - \ \mathcal{W}_{\tau_2} \left(\frac{\mathcal{E}_{\tau_1}}{\mathcal{E}_{\tau_2}} \right) > 0 \ ,$$

where $\mathcal{W}_{\tau_1}$ and $\mathcal{W}_{\tau_2}$ are, respectively, total differentials of a given welfare measure.

[16]The total differential is the sum of a direct effect that a tax increase may have on emissions and of an effect that comes from changes of equilibrium prices in response to the change of the tax rate.

While this test does not involve the solution of an optimal tax problem, there still is a similarity. At a solution to an optimal tax problem with the requirement to reach a given emission target, the ratio $\frac{\mathcal{E}_{\tau_1}}{\mathcal{E}_{\tau_2}}$ has to be proportional to the ratio $\frac{\mathcal{W}_{\tau_1}}{\mathcal{W}_{\tau_2}}$.[17] The test exploits the observation that a lack of proportionality gives us the possibility to reach a higher level of welfare while respecting the emission target. The test can be performed, however, without having to take on board regularity conditions which ensure that a solution to an optimal tax problem is well defined. We only need to evaluate deviations from a given competitive equilibrium allocation. In particular, we can proceed without a need to discuss what welfare weights would look like at an optimal welfare-maximizing allocation.

Welfare implications of policy changes. The following Proposition is the key ingredient for our ability to actually perform the test. It gives the welfare implications of policy changes starting from an arbitrary competitive equilibrium allocation. This characterization of welfare implications takes the form of a sufficient statistics formula; that is, welfare implications can be computed when market outcomes (prices and quantities) are known, when estimates of the elasticities are available that capture how market outcomes change when taxes change, and when welfare weights have been specified.

Proposition 4 *For* $\tau \in \{t_c, t_b, t_g, t_{ce}, t_{be}, t_{ge}\}$,

$$\begin{aligned}
\mathcal{W}_\tau \;=\; & -\sum_j \frac{dq_j^*(\mathcal{T})}{d\tau}\; Cov(\mathbf{g}(\tilde{v}(\cdot),\theta), x_j^*(\cdot)) \\
& + \bar{\mathbf{g}}\; \sum_j (q_j^*(\mathcal{T}) - p_j^*(\mathcal{T}))X_{j\tau}^*(\cdot) \\
& + \frac{dp_c^*(\mathcal{T})}{d\tau_j}\Big((\bar{\mathbf{g}}_{\Pi_c} - \bar{\mathbf{g}})Y_c^*(\cdot) - \sum_j (\bar{\mathbf{g}}_{\Pi_j} - \bar{\mathbf{g}})\mathbf{E}_j[\gamma_j\; r_j^*(\cdot)] \Big) \\
& + \frac{dp_g^*(\mathcal{T})}{d\tau}\; (\bar{\mathbf{g}}_{\Pi_g} - \bar{\mathbf{g}})X_g^*(\cdot) \quad + \frac{dp_b^*(\mathcal{T})}{d\tau}\; (\bar{\mathbf{g}}_{\Pi_b} - \bar{\mathbf{g}})X_b^*(\cdot) \\
& + \bar{\mathbf{g}}\; \mathbf{E}_\theta[T_l'(y_l^*(\cdot,\theta))y_{l\tau}^*(\cdot,\theta)] \\
& + \bar{\mathbf{g}}\; \sum_j t_{je}\; \mathcal{E}_{j\tau}^*(\cdot) \\
& + \sum_j \mathbf{I}(\tau = \tau_{je})(\bar{\mathbf{g}} - \bar{\mathbf{g}}_{\Pi_j})\mathcal{E}_j^*(\cdot)
\end{aligned}$$

where $X_{c\tau}^*$, $X_{g\tau}^*$ *and* $X_{b\tau}^*$ *are total differentials of equilibrium quantities,* $y_{l\tau}^*(\cdot,\theta)$ *is the total differential of equilibrium labour supply for an individual of type* θ *and* $\mathcal{E}_{j\tau}^*$ *is the total differential of equilibrium emissions in sector* j.

The Proposition highlights that the change of a tax has distributive effects and it involves efficiency losses. The distributive effects show up in the terms that relate the social marginal utility of disposable income in a subset of the population to the population average. Efficiency losses, by contrast, are due to the changes of equilibrium quantities. We now provide a more detailed discussion.

[17] See part B of the Appendix for further remarks on optimal policies.

Distributive effects. Suppose that the consumer price of good j goes up when some tax rate changes. When good j is mainly consumed by households with a low social marginal utility of income, so that $\text{Cov}(\mathbf{g}(\tilde{v}(\cdot), \theta), x_j^*(\cdot)) < 0$, then the fact that these households have to pay more for their consumption tends to raise the welfare measure. For instance, when there is a positive correlation of the taste for green goods, as measured by β, and labour income, as measured by ω, and if welfare weights simply depend on disposable income, then this covariance is positive for the brown consumption good and negative for the green consumption good. The signs would be reversed with welfare weights that are "green" in the sense that people with a higher share of the green good in their consumption basket receive more weight than people with a larger share of the brown good.

Alternatively, suppose that the producer price of good j goes down. This tends to increase welfare if those who receive "capital income" from sector j have a welfare weight that is lower than the one of the average person. This is the case when there is a positive correlation between "capital income", as measured by s_j, and labour income and when welfare weights are monotonic in disposable income. Alternatively, with welfare weights that are pro-business in the sense that people with "capital incomes" receive higher weights than the average person, a reduction of producer prices and hence profit margins tends to lower welfare.

The term

$$\sum_j \mathbf{I}(\tau = \tau_{je})(\bar{\mathbf{g}} - \bar{\mathbf{g}}_{\Pi_j})\mathcal{E}_j^*(\cdot)$$

shows that the same logic applies to the evaluation of higher taxes on emissions. If the business owners who have to pay these taxes receive below average weights, this is a plus for welfare, otherwise it is a minus.

Efficiency losses. The term

$$\bar{\mathbf{g}} \sum_j (q_j^*(\mathcal{T}) - p_j^*(\mathcal{T}))X_{j\tau}^*(\cdot)$$

is the general equilibrium analogue to Harberger's famous triangle. Commodity taxes drive a wedge between consumer and producer prices. Therefore they crowd out economic transactions that would be mutually beneficial with lower taxes: $X_{j\tau}^*(\cdot)$ is a measure of the volume of transactions that are lost in the market for good j when taxes change and $q_j^*(\mathcal{T}) - p_j^*(\mathcal{T})$ is a per unit measure of the gains from trade that are lost as a consequence.

The term

$$\bar{\mathbf{g}} \sum_j t_{je}\, \mathcal{E}_{j\tau}^*(\cdot)$$

captures that the volume of emissions changes when taxes change. While a reduction of emissions helps to reach the emissions target, it also implies a loss of tax revenue and hence of welfare.

Finally, changes of the tax system affect consumer prices and thereby the marginal utility of income. This affects earnings incentives on the labour market and hence the tax revenue that comes through the taxation of labour incomes. This is what is picked up by

$$\bar{\mathbf{g}}\ \mathbf{E}_\theta[T_l'(y_l^*(\cdot,\theta))y_{l\tau}^*(\cdot,\theta)]\ .$$

4.1 Sufficient conditions for a uniform price of Carbon

Proposition 4 is based on an arbitrary competitive equilibrium. We now specialize this further and consider the competitive equilibrium that results when climate policy relies on a uniform price of Carbon. We also assume that commodity taxes are uniform. More formally, there is a number $\bar{t}_e$, so that $t_{je} = \bar{t}_e$, for all j. We also let $t_c = t_g = t_b = 0$.[18] Consequently, efficiency losses from commodity taxation vanish

$$\bar{\mathbf{g}}\ \sum_j (q_j^*(\mathcal{T}) - p_j^*(\mathcal{T}))X_{j\tau}^*(\cdot) \quad = \quad 0\ .$$

We now also state the condition of *distributive indifference* in a formal way: Welfare weights are the same for all individuals; i.e. for all θ,

$$\mathbf{g}(\tilde{v}(\beta,q),\theta) = \bar{\mathbf{g}}\ .$$

If this condition holds, only aggregate disposable income matters for welfare. An additional euro of disposable income in the hands of "the rich" is then as valuable as an additional euro in the hands of "the poor". Consequently, all distributive terms disappear from $\mathcal{W}_\tau$ and we are left with

$$\mathcal{W}_\tau \quad = \quad \bar{\mathbf{g}}\ \left(\mathbf{E}_\theta[T_l'(y_l^*(\cdot,\theta))y_{l\tau}^*(\cdot,\theta)] \quad + \quad t_e\,\mathcal{E}_\tau^*(\cdot)\right)\ ,$$

where $\mathcal{E}_\tau^*(\cdot) = \sum_j \ \mathcal{E}_{j\tau}^*(\cdot)$. Now suppose, for a moment, that there is no distortionary income taxation so that $T_l'(y) = 0$, for all levels of y. Under this assumption, we have that

$$\mathcal{W}_\tau \quad = \quad \bar{\mathbf{g}}\,t_e\,\mathcal{E}_\tau^*(\cdot)\ ,$$

and the test for the desirability of the market-based approach yields

$$\mathcal{W}_{\tau_1} \ - \ \mathcal{W}_{\tau_2}\left(\frac{\mathcal{E}_{\tau_1}}{\mathcal{E}_{\tau_2}}\right) = 0\ .$$

[18] Consider the budget constraint of the inner problem: $q_c x_c + q_g x_g + q_b x_b \quad \leq \quad c$. With uniform commodity taxation – i.e. $t_c = t_g = t_b = t$ – this can be written as $p_c x_c + p_g x_g + p_b x_b \quad \leq \quad \frac{c}{1+t}$ where $c = p_w\,y_l - T_l(p_w\,y_l) + s\,\Pi^E + \mathcal{R}^E$. Thus, upon adjusting T_l, s and $\bar{g}$ we can reinterpret the status quo as one that has no commodity taxation at all.

Hence, there is no reason to deviate from a uniform Carbon price.[19] In the presence of distortionary income taxation, however, we have that

$$\mathcal{W}_{\tau_1} \; - \; \mathcal{W}_{\tau_2} \left(\frac{\mathcal{E}_{\tau_1}}{\mathcal{E}_{\tau_2}} \right) =$$

$$\bar{g} \left(\mathbf{E}_\theta [T_l'(y_l^*(\cdot, \theta)) y_{l\tau_1}^*(\cdot, \theta)] - \mathbf{E}_\theta [T_l'(y_l^*(\cdot, \theta)) y_{l\tau_2}^*(\cdot, \theta)] \left(\frac{\mathcal{E}_{\tau_1}}{\mathcal{E}_{\tau_2}} \right) \right) \; .$$

Thus,

$$\mathcal{W}_{\tau_1} \; - \; \mathcal{W}_{\tau_2} \left(\frac{\mathcal{E}_{\tau_1}}{\mathcal{E}_{\tau_2}} \right) = 0 \tag{10}$$

only if

$$\frac{\mathbf{E}_\theta [T_l'(y_l^*(\cdot, \theta)) y_{l\tau_1}^*(\cdot, \theta)]}{\mathbf{E}_\theta [T_l'(y_l^*(\cdot, \theta)) y_{l\tau_2}^*(\cdot, \theta)]} = \frac{\mathcal{E}_{\tau_1}}{\mathcal{E}_{\tau_2}} \; . \tag{11}$$

Thus, to justify a uniform price of Carbon, the various tax instruments under consideration need to be such that (11) holds for any pair τ_1, τ_2. If this property holds, we say for short that there are *proportional fiscal externalities*.[20] To understand the content of this condition, note that any change of the climate policy-mix leads to an adjustment of equilibrium consumer prices $q^*(\cdot)$ and of equilibrium producer prices $p^*(\cdot)$. The change in consumer prices affects the marginal utility of disposable income (by how much depends on the budget share of the various goods, see equation (61) in part D of the Appendix). Therefore it affects earnings incentives and labour supply so that, ultimately, income tax revenue changes. Changes in producer prices, by contrast, imply that firms adjust output, employment and abatement and their demand for emission permits. This in turn has consequences for the government's revenue from Carbon pricing. If, for a given competitive equilibrium, the ratio of these two effects on overall tax revenue is the same for all the instruments that we consider, then we say that fiscal externalities are proportional. The condition can also be more concisely stated as follows: There is a number η, so that, for all $\tau \in \{t_c, t_b, t_g, t_{ce}, t_{be}, t_{ge}\}$,

$$\frac{\mathbf{E}_\theta [T_l'(y_l^*(\cdot, \theta)) y_{l\tau}^*(\cdot, \theta)]}{\mathcal{E}_\tau} \quad = \quad \eta \; ,$$

We then have

$$\mathcal{W}_\tau \quad = \quad \bar{\mathbf{g}} \; (\bar{t}_e + \eta) \; \mathcal{E}_\tau \; ,$$

[19] In Ramsey models in which all taxes are constrained to be linear, it is without loss of generality to assume that one good remains untaxed, and the untaxed good can be taken to be labour income, see e.g. Salanié (2003). Hence, marginal income tax rates are equal to zero. Thus, our analysis implies that, in such a model, there is no reason to deviate from a uniform price on Carbon. See part C of the Appendix for further remarks on Ramsey models of taxation.

[20] We do not take a stance on wether this property is empirically plausible. For the purpose of the current analysis, the condition brings conceptual clarity to the discussion of whether deviations from a uniform Carbon price are desirable.

for all τ, with the implication that (10) holds for all for any pair τ_1, τ_2. The following Proposition summarizes the preceding discussion.

Proposition 5 *Consider the competitive equilibrium induced by a uniform price of Carbon. With distributive indifference and proportional fiscal externalities there are no welfare gains from an admissible deviation.*

With distributive indifference, climate policy is evaluated only from an efficiency point of view. As shown above, if there is no distortionary income tax to begin with, then the reliance on a uniform Carbon price is fully justified. By contrast, if a distortionary income tax is in place then deviations from a uniform price of Carbon become desirable unless fiscal externalities are proportional. If fiscal externalities are not proportional, the desirability of deviations follows from a second best logic in the spirit of Lipsey and Lancaster (1956).

Whether fiscal externalities are (close to) proportional is ultimately an empirical question. It is easy to think of setups in which they are not. Suppose, for instance, that both the green and the brown sector are small relative to the "rest of the economy". Also suppose that taxes in the brown and the green good have an equally small impact on earnings incentives, but differ in their impact on emissions, then it is desirable to have higher taxes in the sector where the impact on emissions is smaller, and less revenue from corrective taxes is lost as a consequence. This would, for instance, be the case when general equilibrium effects are small,[21] and when the green good has (close to) zero emissions. Then, the revenue from carbon pricing in the green sector is less elastic, which, ceteris paribus, makes it desirable to set emission taxes in the green sector higher than in the brown sector.

The extent to which fiscal externalities should matter for climate policy design is related to the discussion about the double-dividend hypothesis.[22] The latter is concerned with a move from a status quo with an insufficient climate policy to a new policy with a more appropriate climate policy. The obvious benefit of such a policy change is the improved design of climate policy. If this climate policy change generates additional revenue for the government (via green taxes or the sale of emission certificates), there is the possibility of a revenue-neutral reform package that involves lower taxes elsewhere. These lower taxes are the second benefit. Here, by contrast, the move is from one climate policy mix that is appropriate – in the sense that the government's emission target is reached – to another one that is also appropriate. Under distributive indifference, such a move comes with welfare gains if fiscal externalities are not proportional.

Proposition 5 gives sufficient conditions for the desirability of a uniform price on Carbon. This raises the question whether these conditions are also necessary. The answer is "no." By Proposition 9 in part A of the Appendix, with heterogeneity only in productive

[21] Below, we spell out conditions under which this is the case.

[22] See Jacobs and de Mooij (2015) for a discussion of the various contributions to that literature.

abilities, a uniform price is desirable even when welfare weights for "the poor" are larger than the welfare weights for the "rich." Moreover, even when there is a non-degenerate joint distribution of preferences for green consumption, "capital incomes" and labour incomes, an empirical application of the sufficient statistics approach may bring the result that the welfare gains of departing from the market-based approach are small. To illustrate this possibility, note that we can write the welfare implication of a tax change as

$$\mathcal{W}_\tau \quad = \quad \mathcal{W}_\tau^{net} \; + \; \bar{\mathbf{g}} \; \sum_j t_{je} \frac{d\mathcal{E}_j^*(\cdot)}{d\tau} \; ,$$

where $\mathcal{W}_\tau^{net}$ is the welfare impact net of its impact on the level of emissions. Using this notation, the condition for a desirability of moving away from a uniform price on Carbon can also be written as

$$\mathcal{W}_{\tau_1}^{net} - \mathcal{W}_{\tau_2}^{net} \left(\frac{\mathcal{E}_{\tau_1}}{\mathcal{E}_{\tau_2}} \right) \quad > \quad 0 \; .$$

It is conceivable that this expression is (close to) zero even though the conditions of distributive indifference and proportional fiscal exteranlities do not hold. Whether or not this is the case can only be answered by bringing the theory to data.

4.2 Equity considerations

We now depart form the assumption of distributive indifference and instead consider welfare weights so that the social marginal utility of disposable income is a decreasing function of disposable income. Again, we consider the competitive equilibrium allocation that results with a uniform tax on emissions, $t_{ce} = t_{be} = t_{ge} =: t_e$, and uniform commodity taxation $t_c = t_b = t_g = 0$. We ask whether a reform towards sector-specific taxes would be welfare-improving. Recall from the previous analysis that such a move is desirable unless

$$\mathcal{W}_{\tau_1}^{net} - \mathcal{W}_{\tau_2}^{net} \left(\frac{\mathcal{E}_{\tau_1}}{\mathcal{E}_{\tau_2}} \right) \quad = \quad 0 \; , \tag{12}$$

holds for any pair $\tau_1, \tau_2 \in \{t_{ce}, t_{ge}, t_{be}, t_c, t_g, t_b\}$.

In the following, we argue that one cannot generally expect condition (12) to be satisfied. To make that point, we look at a special case of the more general model developed in Section 2, the basic Ramsey model of taxation. Specifically, we assume that firms operate with constant returns to scale technologies and fixed emission intensities, i.e. a firm in sector j choose labour inputs l so as to maximize

$$p_j \, l - p_w \, l - t_{je} \, e_{j0} \, l \; .$$

In a competitive equilibrium with $p_w = 1$, producer prices are fixed and given by

$$p_j = 1 + t_{je} \, e_{j0} \; . \tag{13}$$

Moreover, firms make zero profits in equilibrium. Consequently, tax increases are fully passed to consumers,

$$q_j = (1 + t_j)(1 + t_{je}\, e_{j0}) \tag{14}$$

so that there are no general equilibrium effects of taxes on prices: Taxes on goods sold by firms in sector j or taxes on emission permits demanded by firms in sector j affect only the consumer price of good j and no other price.[23] This setup can be interpreted as a limit case of the model in Section 2: When production functions are close to linear and marginal abatement cost functions very steep, then firms are close to operating with constant returns to scale and fixed emission intensities. Under these assumptions the analysis of welfare implications of tax policy changes simplifies considerably.

Lemma 5 *Suppose that equilibrium profits are zero, that producer prices are given by (13) and that consumer prices are given by (14). Then: for* $\tau \in \{t_c, t_b, t_g, t_{ce}, t_{be}, t_{ge}\}$,

$$\begin{aligned}
\mathcal{W}_\tau \;\; = \;\; & -\textstyle\sum_j \frac{dq_j^*(\mathcal{T})}{d\tau}\; Cov(\mathbf{g}(\tilde{v}(\cdot), \theta), x_j^*(\cdot)) \\
& + \bar{\mathbf{g}}\; \textstyle\sum_j (q_j^*(\mathcal{T}) - p_j^*(\mathcal{T})) X_{j\tau}^*(\cdot) \\
& + \bar{\mathbf{g}}\; \mathbf{E}_\theta [T_l'(y_l^*(\cdot, \theta)) y_{l\tau}^*(\cdot, \theta)] \\
& + \bar{\mathbf{g}}\; \textstyle\sum_j t_{je}\, \mathcal{E}_{j\tau}^*(\cdot)\; .
\end{aligned}$$

where

$$\frac{dq_j^*(\mathcal{T})}{d\tau} = \begin{cases} 1 + t_{je}\, e_{j0}, & \textit{if} \quad \tau = t_j\ , \\ (1 + t_j)\, e_{j0}, & \textit{if} \quad \tau = t_{je}\ , \\ 0, & \textit{else}\quad . \end{cases} \tag{15}$$

If we consider deviations from a policy with $t_{ce} = t_{be} = t_{ge} =: t_e$, and $t_c = t_b = t_g = 0$, this expression simplifies further and we are left with

$$\begin{aligned}
\mathcal{W}_\tau \;\; = \;\; & -\textstyle\sum_j \frac{dq_j^*(\mathcal{T})}{d\tau}\, \mathrm{Cov}(\mathbf{g}(\tilde{v}(\cdot), \theta), x_j^*(\cdot)) \\
& + \bar{\mathbf{g}}\; \mathbf{E}_\theta [T_l'(y_l^*(\cdot, \theta)) y_{l\tau}^*(\cdot, \theta)] \\
& + \bar{\mathbf{g}}\; \bar{t}_e\; \mathcal{E}_\tau^*(\cdot)\; .
\end{aligned} \tag{16}$$

Proposition 6 *Suppose that producer prices are given by (13), that consumer prices are given by (14) and that profits are zero in equilibrium. Consider the competitive equilibrium allocation that results with a uniform price on Carbon and without differential commodity taxation. Suppose that fiscal externalities are proportional. Consider two goods* $j, k \in \{c, g, b\}$ *so that*

$$Cov(\mathbf{g}(\cdot), x_k^*(\cdot)) \quad < \quad 0 \quad < \quad Cov(\mathbf{g}(\cdot), x_j^*(\cdot))\; .$$

Welfare goes up when t_{ke} *or* t_k *is increased and when* t_{je} *or* t_j *is decreased.*

[23] Note that without emissions, so that $e_{j0} = 0$, for all j, producer prices would be fixed and equal to 1. This is the canonical setup in the theory of optimal commodity taxation.

The Proposition follows from the observation that, with proportional fiscal externalities,

$$\mathcal{W}^{net}_{\tau_1} - \mathcal{W}^{net}_{\tau_2} \left(\frac{\mathcal{E}_{\tau_1}}{\mathcal{E}_{\tau_2}} \right) \quad = \quad -\frac{dq^*_k}{d\tau_1} \operatorname{Cov}(\mathbf{g}(\cdot), x^*_k(\cdot)) + \frac{dq^*_j}{d\tau_2} \operatorname{Cov}(\mathbf{g}(\cdot), x^*_j(\cdot)) \left(\frac{\mathcal{E}_{\tau_1}}{\mathcal{E}_{\tau_2}} \right) .$$

This expression is positive when, as we assumed, $\operatorname{Cov}(\mathbf{g}(\cdot), x^*_k(\cdot)) < 0 < \operatorname{Cov}(\mathbf{g}(\cdot), x^*_j(\cdot))$, and τ_1 is a tax that raises the consumer price for good k, whereas τ_2 is a tax that raises the consumer price of good j. The interpretation is that good k is mostly consumed by those with low welfare weights ("the rich"), that good j is mostly consumed by those with high welfare weights ("the poor") and that welfare goes up when taxes are increased on the former and reduced on the latter.

Proposition 6 is based on the assumption that fiscal externalities are proportional. It is complemented by the following Proposition which requires no such assumption. The Proposition implies, moreover, that with heterogeneity only in productive abilities, but not in preferences for green versus brown consumption goods, there is neither a case for higher taxes on the brown good, nor for subsidies of the green good, thereby reproducing the insights in part A of the Appendix (see Proposition 9), using the dual approach.

Proposition 7 *Suppose that producer prices are given by (13), that consumer prices are given by (14) and that profits are zero in equilibrium. Suppose that u and χ have the functional forms specified in (7) and (8). Consider the competitive equilibrium allocation that results with a uniform price on Carbon and without differential commodity taxation.*

1. *Suppose that $\mathcal{W}^{net}_{\tau} = 0$ for $\tau \in \{t_c, t_{ce}\}$ and $Cov(\mathbf{g}(\cdot), x^*_j(\cdot)) < Cov(\mathbf{g}(\cdot), x^*_c(\cdot))$. Then $\mathcal{W}^{net}_{\tau} > 0$ for $\tau \in \{t_j, t_{je}\}$ and $\mathcal{W}^{net}_{\tau} < 0$ for $\tau \in \{t_{-j}, t_{-je}\}$.*

2. *Suppose that for all individuals β takes the same value, henceforth denoted by $\bar{\beta}$. Then, for any pair $\tau_1, \tau_2 \in \{t_{ce}, t_{ge}, t_{be}, t_c, t_g, t_b\}$*

$$sgn\, \mathcal{W}^{net}_{\tau_1} \quad = \quad sgn\, \mathcal{W}^{net}_{\tau_2} \; .$$

By part 1. of the Proposition, when the taxes on the unspecific consumption good are (close to) optimal in the sense that the net welfare gains from higher taxes are close to zero, then welfare goes up when taxes are increased on a good that is consumed to a larger extent by "the rich", irrespectively of whether that good is the green good or the brown good. Therefore, a move towards subsidies for the brown good improves welfare when "the poor" tend to have lower values of β.

By Proposition 9) in part A of the Appendix, when there is heterogeneity only in productive abilities, then any departure from a uniform Carbon price implies an inefficiency: There is then a possibility to improve the policy-mix in such a way that everyone benefits. How does this relate to the analysis in this section? Suppose that there is a uniform price in the stats quo and also suppose that there is heterogeneity only in productive abilities. This does not preclude the possibility to increase welfare, say, by increasing emissions taxes on the brown good and decreasing emissions taxes on the unspecific consumption good. Depending on the welfare measure and on fiscal externalities, such a reform can be an improvement over the status quo. The existence of such a reform requires that there is $\tau \in \{t_{ce}, t_{ge}, t_{be}, t_c, t_g, t_b\}$ with $\mathcal{W}_\tau^{net} \neq 0$. Suppose for the sake of the argument that $\mathcal{W}_\tau^{net} > 0$, for some τ. Then by part 2. of Proposition 7, there is a reform that is even better: $\mathcal{W}_\tau^{net} > 0$, for some τ implies $\mathcal{W}_\tau^{net} > 0$, for all τ. Thus, welfare goes up even more if all taxes are raised and the consumer prices of all goods go up. Raising all consumer prices at the same time is, however, equivalent to an increase of the income tax. Thus, when a move to sector-specific taxes is a welfare-improvement, then welfare can be raised even more with an adjustment of the income tax.

5 Concluding remarks

This paper has shown that climate policy is confronted with an equity-efficiency trade-off. A uniform price on Carbon is efficient in the sense that it allows to reach national emission targets at minimal costs. At the same time, deviations to a sector-specific climate policy can be justified by distributive concerns. While such a deviation has an efficiency cost it may improve social welfare. In the presence of non-linear income taxes, a second-best logic may imply, moreover, that deviations from a uniform price can be justified by efficiency considerations.

Relying on a uniform price of Carbon has advantages of simplicity and accountability. Those are not captured by the welfare analysis that is presented in this paper. With a uniform price there is a one-to-one mapping between one policy instrument and one policy goal. It is then very clear what needs to be done when emission targets are missed. The price for Carbon needs to go up.

Still, as suggested by the welfare analysis in this paper, the distributive implications of such an approach may be perceived as unfair. Possibly this is an explanation for the lack of political support and the protests that are spurred by plans for more ambitious climate policies. Reaching emission targets in a politically feasible way may therefore require a sector-specific approach. A more systematic analysis of the conditions under which climate policy can attract majority support is a topic for future research.

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A Benchmark results

We present two benchmark results: Propositions 8 and 9 below clarify conditions under which every Pareto-efficient policy involves a uniform price of Carbon.

A.1 First-best

Let there be a given utility profile $U_0 : \theta \mapsto U_0(\theta)$. We say that an allocation is first best if it is physically feasible and reaches this utility profile with minimal emissions. Thus, a first-best allocation solves the following problem: What has to be chosen are labour supply $y_l : \theta \mapsto y_l(\theta)$ and consumption levels $x_c : \theta \mapsto x_c(\theta)$, $x_b : \theta \mapsto x_b(\theta)$ and $x_g : \theta \mapsto x_g(\theta)$ for the different types of households. In addition, for every sector $j \in \{c, b, g\}$ and every type of firm $\theta_j = (\alpha_j, \gamma_j)$ in that sector, labour inputs and resources devoted to the abatement of emissions need to be chosen. This is captured by the functions $l_j : \theta_j \mapsto l_j(\theta_j)$ and $r_j : \theta_j \mapsto r_j(\theta_j)$. The objective is to minimize

$$\sum_{j\in\{c,b,g\}} \mathcal{E}_j \quad = \quad \sum_{j\in\{c,b,g\}} \mathbf{E}_j \left[\Big(e_{0j} - a_j(r_j(\theta_j)\Big) \alpha_j f_j(l_j(\theta_j)) \right]$$

subject to the following constraints: First, the chosen allocation needs to reach utility profile U_0. Formally, for all θ,

$$u(x_c(\theta), \chi(\beta x_g(\theta), x_b(\theta))) - k(y(\theta), \omega) = U_0(\theta) \ . \tag{17}$$

Second, the labour used up in the production process is bounded from above by the amount that households make available,

$$\sum_{j\in\{c,b,g\}} \mathbf{E}_j \left[l_j(\theta_j) \right] \quad \leq \quad \mathbf{E}_\theta \left[y(\theta) \right] \ . \tag{18}$$

Third, aggregate consumption is bounded by the production sector's (net) output of the various goods. For the unspecific consumption good this requires that

$$\mathbf{E}_\omega[x_c(\omega)] \quad \leq \quad \mathbf{E}_c[\alpha_c f_c(l_c(\theta_c))] - \sum_{j\in\{c,b,g\}} \mathbf{E}_j[\gamma_j r_j(\theta_j)] \ . \tag{19}$$

For the green and the brown good the constraints are, respectively,

$$\mathbf{E}_\omega[x_g(\omega)] \quad \leq \quad \mathbf{E}_g[\alpha_g f_g(l_g(\theta_g))] \quad \text{and} \quad \mathbf{E}_\omega[x_b(\omega)] \quad \leq \quad \mathbf{E}_b[\alpha_b f_b(l_b(\theta_b))] \ . \tag{20}$$

Proposition 8 *At a solution to a first-best problem:*

i) The marginal costs of emission avoidance are equalized: For any $j, k \in \{c, g, b\}$, and any pair $\theta_j = (\alpha_j, \gamma_j)$ and $\theta_k = (\alpha_k, \gamma_k)$,

$$\frac{\gamma_j}{a'_j(r_j(\theta_j))\alpha_j f_j(l_j(\theta_j))} = \frac{\gamma_k}{a'_k(r_k(\theta_k))\alpha_k f_k(l_k(\theta_k))} \, . \tag{21}$$

ii) The marginal rates of substitution between any pair of consumption goods are equalized across households.

iii) The marginal rates of substitution between consumption goods and effort costs are equalized across households.

We omit a formal proof, which can be obtained with standard arguments from a Lagrangean approach. Upon relating the conditions that characterize a first-best allocation to those that characterize a competitive equilibrium allocation we obtain the following Corollary.

Corollary 1 *With sector specific CO2 prices or differential commodity taxation or non-linear income taxation, a competitive equilibrium allocation is not a first best-allocation.*

As is well known, with private information on preferences or abilities, first best allocations that involve redistribution in favor of "the poor" are typically not incentive-compatible. First-best allocations that are incentive-compatible have distributive implications which may be deemed problematic. This is the root of the equity-efficiency trade-off in the Mirrleesian theory of optimal taxation. It is concerned with a second-best problem, welfare-maximization over the set of incentive-compatible allocation. As we will now see, when incentive compatibility constraints need to address only private information in productive abilities, then every-second best allocation is compatible with a uniform price of Carbon.

A.2 Second-best with heterogeneity only in productive abilities

Assumption 1 *Suppose that all individuals have the same preferences over consumption goods, i.e. β is the same for all. Also suppose that all individuals have identical claims on the profits generated in the economy, i.e. s is the same for all. Thus, individuals differ only in their productive abilities ω.*

Under Assumption 1, there is heterogeneity only in productive abilities. Labour earnings and household consumption can therefore more simply be described as functions of ω, rather than as functions of the triple $\theta = (\theta, s, \omega)$. When productive abilities are, moreover, private information incentive compatibility constraints need to be respected. Let

$U_0 : \omega \mapsto U_0(\omega)$ be a given utility profile. Incentive compatibility requires that for every pair ω and ω' in the set of ability types Ω, we have

$$U(\omega) \quad \geq \quad u_0(\omega') - k(y_l(\omega'), \omega) \ , \tag{22}$$

where $u_0 = \omega \mapsto u_0(\omega)$ is the profile of consumption utilities. The second-best problem is otherwise as the first-best problem stated above; i.e. an allocation is chosen to minimize total emissions subject to the constraints of physical feasibility and the requirement to reach a given profile of utilities.

Proposition 9 *Under Assumption 1, at a solution to a second-best problem:*

i) The marginal costs of emission avoidance are equalized: For any $j, k \in \{c, g, b\}$, and any pair $\theta_j = (\alpha_j, \gamma_j)$ and $\theta_k = (\alpha_k, \gamma_k)$,

$$\frac{\gamma_j}{a_j'(r_j(\theta_j))\alpha_j f_j(l_j(\theta_j))} = \frac{\gamma_k}{a_k'(r_k(\theta_k))\alpha_k f_k(l_k(\theta_k))} \ . \tag{23}$$

ii) The marginal rates of substitution between any pair of consumption goods are equalized across households.

Corollary 2 *Under Assumption 1, a competitive equilibrium allocation with sector-specific emission taxes or differential commodity taxation is not a second best-allocation.*

Non-linear income taxation is no impediment for reaching a second-best outcome. While a second-best outcome requires that marginal rates of substitution between consumption goods are not distorted away from the marginal rates of transformation prevailing in the production sector, the marginal rate of substitution between consumption utility and effort costs can be distorted by a non-linear income tax. Second-best allocations under Assumption 1 may therefore involve substantial redistribution.

Assumption 1 is interesting as a benchmark, but it is not empirically plausible. Empirically, income and wealth are correlated and green tastes seem to be more prevalent among “the rich.” Therefore, in the main text, we study on the desirability of a uniform price of Carbon without imposing Assumption 1.

A.3 Proof of Proposition 9

Consider a given utility profile $U_0 : \omega \mapsto U_0(\omega)$, where

$$U_0(\omega) = u_0(\omega) - k(y_{l0}(\omega), \omega) \ , \tag{24}$$

and

$$u_0(\omega) := u(x_{0c}(\omega), \chi(\beta, x_{0g}(\omega), x_{0b}(\omega)))$$

is the consumption utility realized by a type ω-individual in the status quo. In the following we take the function U_0 as given and seek to find that allocation that reaches these utility levels with minimal emissions subject to the economy’s resource constraint and the requirement of incentive compatibility.

Incentive compatibility. By standard arguments, incentive compatibility in the status quo holds if and only if

$$U_0'(\omega) = -k_2(y_{l0}(\omega), \omega) \tag{25}$$

and if the function y_{l0} is non-decreasing. Consequently, for $\underline{\omega} = \text{argmin}\ \Omega$, we have

$$U_0(\omega) = U_0(\underline{\omega}) - \int_{\underline{\omega}}^{\omega} k_2(y_{l0}(n), n) dn$$

and

$$u_0(\omega) = U_0(\omega) - k(y_{l0}(\omega), \omega) \ . \tag{26}$$

Thus, if we take the utility profile $U_0 : \omega \mapsto U_0(\omega)$ and hence also the derivative of this function $U_0' : \omega \mapsto U_0'(\omega)$ as given, then due to (25), we also take the profile $y_{l0} : \omega \mapsto y_{l0}(\omega)$ as given. It then follows from (26) that, with the functions U_0 and y_{l0} given, also the consumption utility profile $u_0 : \omega \mapsto u_0(\omega)$ is given.

Thus, for the problem to reach the status quo utilities with minimal emissions, we can as well assume that $y_{l0} : \omega \mapsto y_{l0}(\omega)$, and hence aggregate labour supply $Y_0 = E_\omega[y_0(\omega)]$, as well as $u_0 : \omega \mapsto u_0(\omega)$ are predetermined. A solution to this problem that respects these constraints will be incentive compatible by construction.

The optimization problem. What has to be chosen are consumption levels $x_c : \omega \mapsto x_c(\omega)$, $x_b : \omega \mapsto x_b(\omega)$ and $x_g : \omega \mapsto x_g(\omega)$ for households that differ in productivity. In addition, for every sector $j \in \{c, b, g\}$ and every type of firm $\theta_j = (\alpha_j, \gamma_j)$ in that sector labour inputs and resources devoted to the abatement of emissions need to be chosen. Formally, the functions $l_j : \theta_j \mapsto l_j(\theta_j)$ and $r_j : \theta_j \mapsto r_j(\theta_j)$ need to be chosen.

The objective is to minimize

$$\sum_{j \in \{c,b,g\}} \mathcal{E}_j \quad = \quad \sum_{j \in \{c,b,g\}} \mathbf{E}_j \left[\Big(e_{0j} - a_j(r_j(\theta_j)) \Big) \alpha_j f_j(l_j(\theta_j)) \right]$$

subject to the following constraints: First, the chosen allocation needs to generate the same consumption utility as the status quo allocation. Formally, for all ω,

$$u(x_c(\omega), \chi(\beta x_g(\omega), x_b(\omega))) = u_0(\omega) \ . \tag{27}$$

Second, the labour used up in the production process is bounded from above by Y_0, the amount that households make available in the status quo,

$$\sum_{j \in \{c,b,g\}} \mathbf{E}_j \left[l_j(\theta_j) \right] \quad \leq \quad Y_0 \ . \tag{28}$$

Third, aggregate consumption is bounded by the production sector's (net) output of the various goods. For the unspecific consumption good this requires that

$$\mathbf{E}_\omega[x_c(\omega)] \quad \leq \quad \mathbf{E}_c[\alpha_c f_c(l_c(\theta_c))] - \sum_{j \in \{c,b,g\}} \mathbf{E}_j[\gamma_j r_j(\theta_j)] \ . \tag{29}$$

For the green and the brown good the constraints are, respectively,

$$\mathbf{E}_\omega[x_g(\omega)] \quad \leq \quad \mathbf{E}_g[\alpha_g f_g(l_g(\theta_g))] \quad \text{and} \quad \mathbf{E}_\omega[x_b(\omega)] \quad \leq \quad \mathbf{E}_b[\alpha_b f_b(l_b(\theta_b))] \ . \tag{30}$$

Solving the problem. Consider the Lagrangean

$$\begin{aligned}
\mathcal{L} \;:=\; & \sum_{j\in\{c,b,g\}} \mathbf{E}_j \left[\Big(e_{0j} - a_j(r_j(\theta_j)\Big)\alpha_j f_j(l_j(\theta_j))\right] \\
& -\mathbf{E}_\omega \left[\mu(\omega)\Big(u_0(\omega) - u(x_c(\omega), \chi(\beta, x_g(\omega), x_b(\omega)))\Big)\right] \\
& -\lambda_l\Big(\sum_{j\in\{c,b,g\}} \mathbf{E}_j\left[l_j(\theta_j)\right] - Y_0\Big) \\
& -\lambda_c\Big(\mathbf{E}_\omega[x_c(\omega)] + \sum_{j\in\{c,b,g\}} \mathbf{E}_j[\gamma_j r_j(\theta_j)] - \mathbf{E}_c[\alpha_c f_c(l_c(\theta_c))]\Big) \\
& -\lambda_g\Big(\mathbf{E}_\omega[x_g(\omega)] - \mathbf{E}_g[\alpha_g f_g(l_g(\theta_g))]\Big) \\
& -\lambda_b\Big(\mathbf{E}_\omega[x_b(\omega)] - \mathbf{E}_b[\alpha_b f_b(l_b(\theta_b))]\Big)
\end{aligned}$$

where $\mu(\omega) := \frac{\nu(\omega)}{\phi(\omega)}$ whith $\nu(\omega)$ the Lagrangean multiplier for the constraint in (27) and ϕ the density associated with the distribution of ω, and λ_l, λ_c, λ_g and λ_b are the multipliers associated with the resource constraints. The Proposition now follows in a straightforward way from an inspection of the first order conditions associated with the maximization of the Lagrangean.

B A Remark on optimal policies

Remember that for any tax τ, we can write

$$\mathcal{W}_\tau \quad = \quad \mathcal{W}_\tau^{net} + \, \bar{\mathbf{g}} \, \sum_j t_{je} \frac{d\mathcal{E}_j^*(\cdot)}{d\tau} \; ,$$

where

$$\bar{\mathbf{g}} \, \sum_j t_{je} \frac{d\mathcal{E}_j^*(\cdot)}{d\tau}$$

is the loss of revenue from higher corrective taxes that is due to behavioral responses, and $\mathcal{W}_\tau^{net}$ gives the welfare implications of higher taxes, without taking that revenue loss into account. Now consider a restricted set of tax instruments: There can be taxes on different commodities t_c, t_g, t_b and there is a uniform tax on emissions, $\bar{t}_e$. Also suppose that t_c, t_g and t_b are set optimal in the sense that

$$\mathcal{W}_\tau^{net} \quad = \quad 0 \; ,$$

for $\tau \in \{t_c, t_g, t_b\}$ and $\bar{t}_e$ is set such that the target level of emissions is reached. This leads to an allocation under which the condition

$$\mathcal{W}_{\tau_1}^{net} - \mathcal{W}_{\tau_2}^{net} \left(\frac{\mathcal{E}_{\tau_1}}{\mathcal{E}_{\tau_2}}\right) \quad = \quad 0 \; , \tag{31}$$

is satisfied, so that, locally, there are no gains from a reform of this tax system. One might debate, whether $\mathcal{W}_\tau^{net} = 0$, for all τ, is the appropriate optimality condition. It is based on an optimization that disregards the implications of climate policy for tax revenue. Taking these revenue implications into account, by contrast, requires to have $\mathcal{W}_\tau = 0$ for all τ. If that's the case, then $\mathcal{W}_\tau^{net} = \bar{g}\ \bar{t}_e\ \mathcal{E}_\tau$, for all $\tau \in \{t_c, t_g, t_b\}$. Again, the implication is that condition (31) is satisfied. Thus, which of these conditions is employed doesn't make a difference for the answer to question, whether, locally, there are gains from a reform the commodity tax system. To complicate matters even more note that a Lagrangean approach to the problem of maximizing welfare subject to the requirement of reaching a given emissions target would yield optimality conditions of the form

$$\mathcal{W}_\tau = \lambda\ \mathcal{E}_\tau\ ,$$

for all τ, where λ is the multiplier associated with the constraint that emissions must not exceed a given target level. Again, the implication is that condition (31) holds for any pair τ_1 and τ_2. This discussion shows that various combinations of policies may satisfy the condition that locally there is no room for improvement. Condition (31) is a necessary condition for optimality, not a sufficient one. The analysis in this paper has stayed away from a characterization of optimal polices and instead focussed on this necessary condition to answer the question whether a uniform price of Carbon leaves room for an improvement of the tax system.

C A remark on Ramsey models of taxation

Ramsey models of taxation are known for the *inverse elasticities rule*. When the government needs to raise revenue with distortionary taxes, it should set higher taxes on goods where behavioral responses are less pronounced. In this paper, there is no revenue requirement of the government but a target level of emissions. Intuitively, on might think that this induces a race between the inverse elasticities logic, whose rationale is to protect gains from trade as much as possible, and the need to reach the emission target. Strong behavioral responses make it easy to reduce emissions and this may be a reason to tax goods with high elasticities more. The analysis in this paper shows that such an intuition would be misguided. With distributive indifference and without labour income taxation (a frequent assumption in Ramsey models of taxation), Proposition 4 implies that, for any $\tau \in \{t_c, t_b, t_g, t_{ce}, t_{be}, t_{ge}\}$,

$$\begin{aligned} \mathcal{W}_\tau \;=\; & +\bar{\mathbf{g}}\ \textstyle\sum_j (q_j^*(\mathcal{T}) - p_j^*(\mathcal{T})) X_{j\tau}^*(\cdot) \\ & +\bar{\mathbf{g}}\ \textstyle\sum_j t_{je}\ \mathcal{E}_{j\tau}^*(\cdot)\ , \end{aligned} \tag{32}$$

where the term in the first line is a measure of the economic surplus that is lost when taxes drive a wedge between consumer and producer prices and the term in the second line gives the implications of higher taxes on the revenue generated by corrective taxes. At a hypothetical status quo without differential commodity taxation and a uniform Carbon tax, the term in the first line vanishes with the implication that condition

$$\mathcal{W}^{net}_{\tau_1} - \mathcal{W}^{net}_{\tau_2} \left(\frac{\mathcal{E}_{\tau_1}}{\mathcal{E}_{\tau_2}} \right) \quad = \quad 0 \, ,$$

is satisfied for any pair (τ_1, τ_2), implying that there is no reason to deviate from a uniform price of Carbon. Note that both the desire to protect the economic surplus that is generated in competitive markets and the response of emissions to taxes are part of the analysis that leads to this conclusion. Still, the elasticities logic does not play out.

Here, the elasticities logic would play out in an optimal tax formula that applies when deviations from the uniform price are desirable. Away from the benchmark of uniform commodity taxation $\bar{\mathbf{g}} \ \sum_j (q^*_j(\mathcal{T}) - p^*_j(\mathcal{T})) X^*_{j\tau}(\cdot)$, becomes part of the equation that is used to evaluate the welfare effects of policy changes.

The treatment of tax revenue is an important difference between analyses in the tradition of Ramsey (1927) and the analysis presented in this paper. In Ramsey's analysis tax revenue is extracted from individuals, here tax revenue is rebated lump sum and therefore a source of welfare for individuals. Here, with distributive indifference, it is harmful if tax revenue is generated through distortionary commodity taxes (the first term in the right-hand side of (32) is unambiguously negative), and, moreover, there is the possibility to refrain from the use of such taxes. In Ramsey models, by contrast, for exogenous reasons, tax revenue has to be generated through distortionary taxes.

D Proofs

D.1 Proof of Lemma 2

The first order condition of the outer problem can be rewritten as

$$(e_0 - a(r)) \alpha f'_j(l^*) \quad = \quad \frac{p_j \, \alpha f'(l^*) - p_w}{t_{je}} \, ,$$

or, using that $f_j(l) = \frac{1}{1-\frac{1}{\sigma_j}} l^{1-\frac{1}{\sigma_j}}$,

$$(e_0 - a(r)) \alpha f_j(l^*) \frac{1-\frac{1}{\sigma_j}}{l^*} \quad = \quad \frac{p_j \, \alpha f_j(l^*) \frac{1-\frac{1}{\sigma_j}}{l^*} - p_w}{t_{je}} \, .$$

Hence

$$\mathbf{e}^* \quad = \quad \frac{p_j \, \alpha f(l^*) - p_w l^* \left(1 - \frac{1}{\sigma_j}\right)^{-1}}{t_{je}}$$

Note that $\mathbf{e}^*$ is increasing in l^*: The derivative of the right hand side with respect to l^* equals

$$\frac{1}{t_{je}}\left(p_j\alpha f_j'(l^*)-p_w\left(1-\frac{1}{\sigma_j}\right)^{-1}\right)>\frac{1}{t_{je}}\left(p_j\alpha f_j'(l^*)-p_w\right)>0$$

Thus, emissions go up if (i) p_j increases, (ii) p_w decreases, or (iii) t_{je} decreases. The direct effect and the effect via l^* have the same sign. All other parameters affect emissions only via l^*. In any case, the effect on l^* has the same sign as the effect on $\mathbf{e}^*$. □

D.2 Proof of Lemma 3

Inserting $\mathcal{R}(p,\mathcal{T})$ for $\mathcal{R}^E$ in the right hand side of equation (6) turns this equation into a fixed point equation that can also be written as

$$G(\rho):=\rho\ ,$$

where

$$\begin{aligned}G(\rho) &= t_c\,p_c\,X_c(\Pi^E,\rho,q,p_w,T_l)\\ &\quad+t_g\,p_g\,X_g(\Pi^E,\rho,q,p_w,T_l)\\ &\quad+t_b\,p_b\,X_b(\Pi^E,\rho,q,p_w,T_l)\\ &\quad+\mathbf{E}_\theta\Big[T_l(p_w\,y_l^*(\theta,\Pi^E,\rho,q,p_w,T_I))\Big]\\ &\quad+\textstyle\sum_{j\in\{c,b,g\}}\ t_{je}\,\mathcal{E}_j(p^j,t_{je})\ .\end{aligned}$$

<u>*Step 1.*</u> We first show that, for all ρ, $G'(\rho)<1$. To see this, note that $G(\rho)$ can also be written as a sum of the tax revenue due to individuals and the sum of tax revenue due to firms.

$$G(\rho)=G_I(\rho)+G_F\ ,\quad \text{where}\quad G_F=\sum_{j\in\{c,b,g\}}t_{je}\,\mathcal{E}_j(p^j,t_{je})$$

does not depend on ρ and

$$\begin{aligned}G_I(\rho) &:= t_c\,p_c\,X_c(\Pi^E,\rho,q,p_w,T_l)\\ &\quad+t_g\,p_g\,X_g(\Pi^E,\rho,q,p_w,T_l)\\ &\quad+t_b\,p_b\,X_b(\Pi^E,\rho,q,p_w,T_l)\\ &\quad+\mathbf{E}_\theta\Big[T_l(p_w\,y_l^*(\theta,\Pi^E,\rho,q,p_w,T_l))\Big]\ .\end{aligned}$$

can also be written as

$$G_I(\rho)\ :=\ \mathbf{E}_\theta\Big[T(\theta,\rho)\Big]\ ,$$

where

$$T(\theta,\rho)\ =\ T_l(p_w\,y_l^*(\theta,\Pi^E,\rho,q,p_w,T_l))+\sum_{j=c,g,b}t_j\,p_j\,x_j^*(\beta,c^*(\theta,\Pi^E,\rho,\cdot),q)$$

are the tax payments of a type θ-individual, interpreted as a function of exogenous tax revenue ρ, holding fixed the economy's price and tax system. From the individual's budget constraint we have that

$$C(\rho, \theta) + T(\theta, \rho) = \mathcal{I}(\theta, \rho) + \rho \ , \qquad (33)$$

where

$$\mathcal{I}(\theta, \rho) = p_w \ y_l^*(\theta, \Pi^E, \rho, p_w, q, T_l) - T_l(y_l^*(\theta, \Pi^E, \rho, p_w, q, T_l)) + s \ \Pi(p, p_w, t_e)$$

is the sum of the individual's net labour and "capital income" and

$$\begin{aligned} C(\rho, \theta) &= p_c \ x_c^*(\beta, c^*(\theta, \Pi^E, \rho, \cdot), q_x) \\ &\quad + p_g \ x_g^*(\beta, c^*(\theta, \Pi^E, \rho, \cdot), q_x) \\ &\quad + p_b \ x_b^*(\beta, c^*(\theta, \Pi^E, \rho, \cdot), q_x) \end{aligned}$$

are the individual's net expenditures on consumption goods. Equation (33) implies that

$$T_\rho(\theta, \rho) = 1 - C_\rho(\theta, \rho) + \mathcal{I}_\rho(\theta, \rho) \ .$$

Form the assumption that all goods are normal goods it follows that $C_\rho(\theta, \rho) > 0$. The fact that the marginal utility of disposable income is constant, see Lemma 1, can be shown to imply that y_l^* does not depend on ρ, with the consequence that $\mathcal{I}_\rho(\theta, \rho) = 0$. Thus, we have that

$$T_\rho(\theta, \rho) < 1 \ ,$$

and as a consequence

$$G'(\rho) \quad = \quad \mathbf{E}_\theta \Big[T_\rho(\theta, \rho) \Big] \quad < \quad 1 \ .$$

Step 2. Under the given assumptions G is a continuous function on a bounded domain $\rho \in [0, \bar{R}]$. By Brouwer's fixed point theorem, there is a solution to the equation $G(\rho) = \rho$.

Step 3. It remains to be shown that this solution is unique. Step 1 implies that $G(\rho) - \rho$ is a decreasing function. Moreover $G(\rho) \geq 0$, with $G(0) = 0$ indicating that taxes are prohibitive so that no tax revenue is collected and $G(0) > 0$ indicating that there is positive tax revenue even if none of that revenue is rebated lump sum and individuals spend only their labour and "capital" incomes. If $G(0) = 0$ then $\rho = 0$ is the unique solution to the fixed point equation. If $G(0) > 0$ there is a unique $\rho > 0$ solving the fixed point equation $G(\rho) = \rho$. □

D.3 Proof of Lemma 4

The labour market clearing condition can be equivalently written as

$$p_w \left[Y_l(\Pi^E, \mathcal{R}^E, q, p_w, T_l) - L(p, t_e)\right] = 0 \ ,$$

or, as

$$\mathbf{E}_\theta \Big[p_w y_l^*(\cdot)\Big] - p_w \sum_j \mathbf{E}_j \Big[l^*(\cdot)\Big] = 0 \ .$$

Using the individuals' budget constraints and the definition of profits, this can be equivalently written as

$$\begin{aligned}
&\mathbf{E}_\theta \Big[p_c(1+t_c)x_c^*(\cdot) + p_g(1+t_g)x_g^*(\cdot) + p_b(1+t_b)x_b^*(\cdot) + T_l(p_w y_l^*(\cdot)) - s\Pi^E - \mathcal{R}^E\Big] \\
&- \textstyle\sum_j \mathbf{E}_j \Big[p_j \alpha f_j(l^*(\cdot)) - t_{je}\, \mathbf{e}_j^*(\cdot) - \gamma r^*(\cdot) - \pi^j(\cdot)\Big] \\
&= 0 \ .
\end{aligned}$$

This in turn is equivalent to

$$\begin{aligned}
&\mathcal{R} - \mathcal{R}^E \quad + \quad \Pi - \Pi^E \\
&+ p_c \Big[X_c(\Pi^E, \mathcal{R}^E, q, p_w, T_l) + R(p, t_e) - Y_c(p_c, p_w, t_{ce})\Big] \\
&+ p_g \Big[X_g(\Pi^E, \mathcal{R}^E, q, p_w, T_l) - Y_g(p_g, p_c, p_w, t_{ge})\Big] \\
&+ p_b \Big[X_b(\Pi^E, \mathcal{R}^E, q, p_w, T_l) - Y_b(p_b, p_c, p_w, t_{be})\Big] \\
&= 0 \ .
\end{aligned}$$

□

D.4 Proof of Proposition 1

Choice of the numeraire. By Lemma 4, labour market clearing is implied when the three goods markets clear. Thus, to prove existence and uniqueness we can focus without loss of generality on the goods markets. There are four prices that enter the three goods-market clearing conditions. Henceforth, and without loss of generality, we set the wage rate p_w equal to 1.

D.4.1 Firm Behavior and its implications for supply and demand

As explained above, the profit-maximization problem of a firm in sector j can be decomposed in an inner and an outer problem. The inner problem is to choose an investment into emission avoidance r for a given level of employment l and hence a given level of output. The outer problem then is to choose the level of labour demand. We recall this decomposition here as it is useful to determine how the firms' supply and its demand of the unspecific consumption good depend on the economy's price system.

The inner problem. Given l, choose r to maximize

$$-t_{je}\,(e_{j0} - a_j(r))\alpha f_j(l) - p_c\,\gamma\,r$$

The solution to this problem is denoted by $r^*(l, p_c, t_{je}, \gamma)$. It is straightforward to verify that r^* is increasing in l and t_{je} and decreasing in p_c and γ.

The outer problem. The outer problem is to choose l to maximize

$$p_j\;\alpha\;f_j(l) - t_{je}(e_{j0} - a_j(r^*(l,\cdot)))\alpha f_j(l) - p_c\,\gamma\,r^*(l,\cdot)\;.$$

We denote the solution to this problem by $l^*(p_j, p_c, t_{je}, \gamma, \alpha)$. It is straightforward to verify that l^* is increasing in p_j. The complementarity of the investment and the labour choice implies, moreover, that l^* is decreasing in p_c and γ, as r^* is decreasing in these variables.

Implications. (i) Holding fixed p_c, the supply in the green sector increases in p_g and the supply of the brown sector increases in p_b. (ii) Every sector's demand of the unspecific consumption good for investment purposes decreases in p_c. (iii) The net supply of the unspecific consumption good (= supply minus own demand for investment purposes) is increasing in p_c.

D.4.2 Consumption demand with Coub Douglas and CES preferences

The inner problem. With Cobb Douglas and CES preferences, the inner problem can be written as: Choose z_c, z_g and z_b to maximize

$$z_c^{1-\nu}\left(\beta_g^{1-\varepsilon_X} + s_b^{1-\varepsilon_X}\right)^{\frac{\nu}{1-\varepsilon_X}}$$

subject to

$$q_c\;z_c + q_g\;z_g + q_b\;z_b = 1\;.$$

The solution is

$$\mathcal{Z}_c^*(q_c) = \frac{1-\nu}{q_c}\;,\quad z_g^*\left(\beta, \frac{q_b}{q_g}\right) = \frac{\nu}{q_g}\,\alpha_g\left(\beta, \frac{q_b}{q_g}\right)\quad\text{and}\quad \mathcal{Z}_b^*\left(\beta, \frac{q_b}{q_g}\right) = \frac{\nu}{q_b}\,\alpha_b\left(\beta, \frac{q_b}{q_g}\right)\;,$$

where

$$\alpha_g\left(\beta, \frac{q_b}{q_g}\right) := \frac{\beta^{\frac{1}{\varepsilon_X}}\left(\frac{q_b}{q_g}\right)^{\frac{1-\varepsilon_X}{\varepsilon_X}}}{1+\beta^{\frac{1}{\varepsilon_X}}\left(\frac{q_b}{q_g}\right)^{\frac{1-\varepsilon_X}{\varepsilon_X}}}\quad\text{and}\quad \alpha_b\left(\beta, \frac{q_b}{q_g}\right) := 1 - \alpha_g\left(\beta, \frac{q_b}{q_g}\right)\;.$$

Thus, a fraction $1-\nu$ of disposable income is spent on the unspecific consumption good, a fraction $\nu\;\alpha_g\left(\beta, \frac{q_b}{q_g}\right)$ is spent on the green consumption good and a fraction $\nu\;\alpha_g\left(\beta, \frac{q_b}{q_g}\right)$ is spent on the brown consumption good.

The indirect utility $\tilde{v}(\beta, q)$ associated with a solution to the inner problem is obtained by inserting z_c^*, z_g^* and z_b^* into the objective of the inner problem. It follows from Roy's identity that $\tilde{v}(\beta, q)$ is decreasing in q_c, q_g and q_b.

The outer problem. The problem is to choose c and y to maximize

$$c\,\tilde{v}(\beta, q) - k(y_l, \omega) \quad \text{s.t.} \quad c = y_l - T_l(y_l) + s\Pi^E + \mathcal{R}^E \; .$$

Note that the utility maximizing earnings level does neither depend on Π^E nor on $\mathcal{R}^E$. Thus, we can write $y_l^*(\tilde{v}(\beta, q), \omega, T_l)$ and

$$c^*(\tilde{v}(\beta, q), \omega, T_l, s\Pi^E + \mathcal{R}^E) = n_l(y_l^*(\tilde{v}(\beta, q), \omega, T_l) + s\Pi^E + \mathcal{R}^E,$$

where we refer to $n_l(y_l) := y_l - T_l(y_l)$ as the net labour income function. Note that c^* is a decreasing function of $\tilde{v}$ and hence a decreasing function of q_c, q_g and q_b.

Consumption demand. Individual demand for the unspecific consumption good is given by

$$x_c^*(\cdot) = \frac{1-\nu}{q_c}\; c^*(\cdot) \; .$$

Individual demand for the green good is given by

$$x_g^*(\cdot) = \frac{\nu}{q_g}\; \alpha_g \left(\beta, \frac{q_b}{q_g}\right)\; c^*(\cdot) \; ,$$

and individual demand for the brown good is given by

$$x_b^*(\cdot) = \frac{\nu}{q_b}\; \alpha_b \left(\beta, \frac{q_b}{q_g}\right)\; c^*(\cdot) \; .$$

Aggregate demand can therefore be written as

$$X_c(q_c, q_g, q_b, \cdot) = \frac{1-\nu}{q_c}\; \mathbf{E}_\theta \left[c^*(\cdot)\right] \; ,$$

$$X_g(q_c, q_g, q_b, \cdot) = \frac{\nu}{q_g}\; \mathbf{E}_\theta \left[\alpha_g \left(\beta, \frac{q_b}{q_g}\right)\; c^*(\cdot)\right]$$

and

$$X_b(q_c, q_g, q_b, \cdot) = \frac{\nu}{q_b}\; \mathbf{E}_\theta \left[\alpha_b \left(\beta, \frac{q_b}{q_g}\right)\; c^*(\cdot)\right] \; .$$

We define the corresponding excess demand functions as

$$\mathcal{Z}^c(p_c, p_g, p_b, \cdot) \quad := \quad X_c((1+t_c)p_c, (1+t_g)p_g, (1+t_b)p_b, \cdot) - Y_c(p_c, t_{ce}) \; ,$$

$$\mathcal{Z}^g(p_c, p_g, p_b, \cdot) \quad := \quad X_g((1+t_c)p_c, (1+t_g)p_g, (1+t_b)p_b, \cdot) - Y_g(p_g, p_c, t_{ge}) \; ,$$

and

$$\mathcal{Z}^b(p_c, p_g, p_b, \cdot) \quad := \quad X_b((1+t_c)p_c, (1+t_g)p_g, (1+t_b)p_b, \cdot) - Y_b(p_b, p_c, t_{ge}) \; .$$

D.4.3 Implications of ν "small"

It follows from Roy's identity that, for any β,

$$\frac{\partial \tilde{v}(\beta, q)}{\partial q_b} = -\tilde{v}(\beta, q)\ \mathcal{Z}_b^*\left(\beta, \frac{q_b}{q_g}\right) \quad \text{and} \quad \frac{\partial \tilde{v}(\beta, q)}{\partial q_b} = -\tilde{v}(\beta, q)\ \mathcal{Z}_g^*\left(\beta, \frac{q_b}{q_g}\right)$$

With

$$z_g^*\left(\beta, \frac{q_b}{q_g}\right) = \frac{\nu}{q_g}\ \alpha_g\left(\beta, \frac{q_b}{q_g}\right) \quad \text{and} \quad \mathcal{Z}_b^*\left(\beta, \frac{q_b}{q_g}\right) = \frac{\nu}{q_b}\ \alpha_b\left(\beta, \frac{q_b}{q_g}\right)\ ,$$

it follows that

$$\lim_{\nu \to 0} \frac{\partial \tilde{v}(\beta, q)}{\partial q_b} = 0 \quad \text{and} \quad \lim_{\nu \to 0} \frac{\partial \tilde{v}(\beta, q)}{\partial q_b} = 0\ . \tag{34}$$

Using L'Hospital's rule one can show, moreover, that, for all β,

$$\lim_{\nu \to 0} \tilde{v}(\beta, q) = \frac{1}{q_c}\ . \tag{35}$$

Thus, when the budget shares of the green and the brown good become small, then (i) indirect utility no longer depends on the prices of these goods and (ii) their impact on individual welfare vanishes.

Note that the solution to the outer problem $c^*(\cdot)$ depends continuously on $\tilde{v}(\beta, q)$, i.e. on the marginal utility of income. Using the chain rule, a change of the prices for the brown and the green good affect $c^*(\cdot)$ via

$$\frac{\partial c^*(\cdot)}{\partial \tilde{v}(\cdot)}\ \frac{\partial \tilde{v}(\beta, q)}{\partial q_b}$$

and

$$\frac{\partial c^*(\cdot)}{\partial \tilde{v}(\cdot)}\ \frac{\partial \tilde{v}(\beta, q)}{\partial q_b}$$

It follows from (34) that these expressions vanish as $\nu \to 0$. The following Lemma is implied by these observations.

Lemma 6 *Let $\rho_g^{c^*}(\beta, \cdot)$ be the elasticity of $c^*(\cdot)$ with respect to q_g for an individual of type β. Analogously, let $\rho_b^{c^*}(\beta, \cdot)$ be the elasticity of c^* with respect to q_b for an individual of type β. For all β,*

$$\lim_{\nu \to 0}\ \rho_g^{c^*}(\beta, \cdot) = 0 \quad \textit{and} \quad \lim_{\nu \to 0}\ \rho_b^{c^*}(\beta, \cdot) = 0 \tag{36}$$

As an implication also

$$\lim_{\nu \to 0}\ \rho_g^{C^*}(\cdot) = 0 \quad \textit{and} \quad \lim_{\nu \to 0}\ \rho_b^{C^*}(\cdot) = 0 \tag{37}$$

where $\rho_g^{C^*}$ and $\rho_b^{C^*}$, are, respectively, the elasticities of $C^*(\cdot) := E_\theta[c^*(\cdot)]$ with respect to q_g and q_b. Furthermore, for $j \in \{b, g\}$

$$\lim_{\nu \to 0} \frac{\partial \mathcal{Z}^c(\cdot)}{\partial p_j} = 0 \tag{38}$$

and

$$\lim_{\nu \to 0} \frac{\partial X^g(\cdot)}{\partial p_j} = \frac{\nu}{q_g} \mathbf{E}_\theta \left[\left(\frac{\partial}{\partial p_j} \alpha_g \left(\beta, \frac{q_b}{q_g} \right) \right) c^*(\cdot) \right] \tag{39}$$

and

$$\lim_{\nu \to 0} \frac{\partial X^b(\cdot)}{\partial p_j} = \frac{\nu}{q_g} \mathbf{E}_\theta \left[\left(\frac{\partial}{\partial p_j} \alpha_b \left(\beta, \frac{q_b}{q_g} \right) \right) c^*(\cdot) \right] \tag{40}$$

Moreover,

$$\lim_{\nu \to 0} c^*(\tilde{v}(\beta, q), \omega, T_l, s\Pi^E + \mathcal{R}^E) = c^* \left(\frac{1}{q_c}, \omega, T_l, s\Pi^E + \mathcal{R}^E \right) ,$$

so that, in the limit, labour supply and disposable income – i.e. the solutions to the outer problem – do not depend on the prices of the green and the brown consumption good.

Lemma 7 *There is $\bar{\nu}$ so that $\nu < \bar{\nu}$ implies that*

$$\mathcal{Z}^g_{p_g} \mathcal{Z}^b_{p_b} > \mathcal{Z}^g_{p_b} \mathcal{Z}^b_{p_g} , \tag{41}$$

where $\mathcal{Z}^g_{p_g}$ and $\mathcal{Z}^g_{p_b}$ are the derivatives of the excess demand function for the green good with respect to p_g and p_b. $\mathcal{Z}^b_{p_g}$ and $\mathcal{Z}^b_{p_b}$ are analogously defined.

Proof. For convenience, we interpret α_g and α_b as functions of $\tilde{\beta}\left(\frac{1+t_b}{1+t_g}\frac{p_b}{p_g}\right)$, where $\tilde{\beta} = \beta^{\frac{1}{1-\varepsilon_X}}$. We denote by $\alpha_g'(\cdot) > 0$ the derivative of α_g with respect to this expression. We define $\alpha_b'(\cdot) < 0$ analogously. Thus,

$$\mathcal{Z}^g(p_c, p_g, p_b, \cdot) \quad := \quad \frac{\nu}{(1+t_g)p_g} A_g(p_c, p_g, p_b) - Y_g(p_g, p_c, t_{ge}) ,$$

where

$$A_g(p_c, p_g, p_b) := \mathbf{E} \left[\alpha_g \left(\tilde{\beta} \left(\frac{1+t_b}{1+t_g} \frac{p_b}{p_g} \right) \right) c^*(\tilde{v}(\beta, t, p), \cdot) \right] .$$

Analogously,

$$\mathcal{Z}^b(p_c, p_g, p_b, \cdot) \quad := \quad \frac{\nu}{(1+t_b)p_b} A_b(p_c, p_g, p_b) - Y_b(p_b, p_c, t_{be}) ,$$

where

$$A_b(p_c, p_g, p_b) := \mathbf{E} \left[\alpha_b \left(\tilde{\beta} \left(\frac{1+t_b}{1+t_g} \frac{p_b}{p_g} \right) \right) c^*(\tilde{v}(\beta, t, p), \cdot) \right] .$$

and $\tilde{v}(\beta, t, p)$ is a shorthand for $\tilde{v}(\beta, (1+t_c)p_c, (1+t_g)p_g, (1+t_b)p_b)$. Straightforward computations yield

$$\mathcal{Z}^g_{p_g} \quad = \quad -\frac{\nu}{(1+t_g)\,p_g^2}\mathbf{E}\left[\alpha_g(\cdot)\,c^*(\cdot)(1+\rho_{\alpha_g}-\rho_g^{c^*}(\cdot))\right] \;-\; Y^g_{p_g}(\cdot)$$

where ρ_{α_g} is the elasticity of $\alpha_g(\cdot)$ with respect to $\tilde{\beta}\left(\frac{1+t_b}{1+t_g}\frac{p_b}{p_g}\right)$.

$$\mathcal{Z}^g_{p_b} \quad = \quad -\frac{\nu}{(1+t_g)\,p_g\,p_b}\mathbf{E}\left[\alpha_g(\cdot)\,c^*(\cdot)(\rho_{\alpha_g}-\rho_b^{c^*}(\cdot))\right]$$

Moreover,

$$\mathcal{Z}^b_{p_b} \quad = \quad -\frac{\nu}{(1+t_b)\,p_b^2}\mathbf{E}\left[\alpha_b(\cdot)\,c^*(\cdot)(1+\rho_{\alpha_b}-\rho_b^{c^*}(\cdot))\right] \;-\; Y^b_{p_b}(\cdot)$$

and

$$\mathcal{Z}^b_{p_g} \quad = \quad -\frac{\nu}{(1+t_b)\,p_g\,p_b}\mathbf{E}\left[\alpha_b(\cdot)\,c^*(\cdot)(\rho_{\alpha_b}-\rho_g^{c^*}(\cdot))\right] \;,$$

where ρ_{α_b} is the elasticity of $\alpha_b(\cdot)$ with respect to $\tilde{\beta}\left(\frac{1+t_b}{1+t_g}\frac{p_b}{p_g}\right)$. We define elasticities so that they are positive. Hence,

$$\rho_{\alpha_b} := -\frac{\alpha_b'(\cdot)}{\alpha_b(\cdot)}\,\tilde{\beta}\left(\frac{1+t_b}{1+t_g}\frac{p_b}{p_g}\right) \;.$$

Given these expressions, one can compute $\mathcal{Z}^g_{p_g}\mathcal{Z}^b_{p_b}$ and $\mathcal{Z}^g_{p_b}\mathcal{Z}^b_{p_g}$. It is straightforward to verify that a sufficient condition for

$$\mathcal{Z}^g_{p_g}\mathcal{Z}^b_{p_b} > \mathcal{Z}^g_{p_b}\mathcal{Z}^b_{p_g} \;,$$

to hold is that

$$\begin{aligned}
&\mathbf{E}\left[\alpha_g(\cdot)\,c^*(\cdot)\right]\,\mathbf{E}\left[\alpha_b(\cdot)\,c^*(\cdot)\right] \\
&+\mathbf{E}\left[\alpha_g(\cdot)\,c^*(\cdot)\right]\mathbf{E}\left[\alpha_b(\cdot)\,c^*(\cdot)(\rho_{\alpha_b}-\rho_b^{c^*}(\cdot))\right] \\
&+\mathbf{E}\left[\alpha_b(\cdot)\,c^*(\cdot)\right]\mathbf{E}\left[\alpha_g(\cdot)\,c^*(\cdot)(\rho_{\alpha_g}-\rho_g^{c^*}(\cdot))\right] \\
&\geq 0 \;.
\end{aligned}$$

By Lemma 6, $\rho_g^{c^*}(\beta,\cdot)$ and $\rho_b^{c^*}(\beta,\cdot)$ vanish for $\nu \to 0$. Hence, for ν sufficiently small, this inequality holds. □

D.4.4 Existence and uniqueness

For the proof of existence and uniqueness, we let $\tilde{v}(\beta, t, p)$ be a shorthand for $\tilde{v}(\beta, (1+t_c)p_c, (1+t_g)p_g, (1+t_b)p_b)$. We also use the shorthands

$$A_g(p_c, p_g, p_b) := \mathbf{E}\left[\alpha_g\left(\beta, \frac{(1+t_b)p_b}{(1+t_g)p_g}\right)\; c^*(\tilde{v}(\beta, t, p), \cdot)\right] \;,$$

$$A_b(p_c, p_g, p_b) := \mathbf{E}\left[\alpha_b\left(\beta, \frac{(1+t_b)p_b}{(1+t_g)p_g}\right)\ c^*(\tilde{v}(\beta, t, p), \cdot)\right] ,$$

and

$$A_c(p_c, p_g, p_b) := \mathbf{E}\left[\ c^*(\tilde{v}(\beta, t, p), \cdot)\right] .$$

To show existence and uniqueness we find it convenient to use the functions

$$Z^g(p_c, p_g, p_b) := \frac{\nu}{p_g} - \frac{(1+t_g)Y_g(p_g, p_c, t_{ge})}{A_g(p_c, p_g, p_b)} ,$$

$$Z^b(p_c, p_g, p_b) := \frac{\nu}{p_b} - \frac{(1+t_b)Y_b(p_g, p_c, t_{be})}{A_b(p_c, p_g, p_b)} ,$$

and

$$Z^c(p_c, p_g, p_b) := \frac{1-\nu}{p_c} - \frac{(1+t_c)Y_c^{net}(p_c, p_g, p_b, t_e)}{A_c(p_c, p_g, p_b)} ,$$

where Y_c^{net} is the production sector's net supply of the unspecific consumption goods. Existence and uniqueness follow from the observations below. Market clearing requires that

$$Z^g(p_c, p_g, p_b) \quad = \quad Z^b(p_c, p_g, p_b) \quad = \quad Z^c(p_c, p_g, p_b) \quad = \quad 0 .$$

Observation 1. Given p_c and p_b, the excess demand function Z^g is decreasing in p_g and there is a unique value of p_g so that $Z^g(p_c, p_g, p_b) = 0$.

Observation 2. Given p_c and p_g, the excess demand function Z^b is decreasing in p_b and there is a unique value of p_b so that $Z^b(p_c, p_g, p_b) = 0$.

Observation 3. By Lemma 6, if ν is sufficiently small, then the sign of

$$\frac{d}{dp_j}\left[\alpha_b\left(\beta, \frac{(1+t_b)p_b}{(1+t_g)p_g}\right)\ c^*(\tilde{v}(\beta, (1+t_c)p_c, (1+t_g)p_g, (1+t_b)p_b), \cdot)\right]$$

is equal to the sign of

$$\frac{d}{dp_j}\alpha_b\left(\beta, \frac{(1+t_b)p_b}{(1+t_g)p_g}\right) ,$$

for $p_j \in \{p_g, p_b\}$.

Observation 4. By Lemma 6, if ν is sufficiently small, then the sign of

$$\frac{d}{dp_j}\left[\alpha_g(\beta, (1+t_b)p_b, (1+t_g)p_g)\ c^*(\tilde{v}(\beta, (1+t_c)p_c, (1+t_g)p_g, (1+t_b)p_b), \cdot)\right]$$

is equal to the sign of

$$\frac{d}{dp_j}\alpha_g(\beta, (1+t_b)p_b, (1+t_g)p_g) ,$$

for $p_j \in \{p_g, p_b\}$.

Observation 5. (Gross Substitutes.) If ν is sufficiently small, then Z^g is increasing in p_b and Z^b is increasing in p_g.

Observation 6. (Single crossing condition.) Fix p_c. Consider a p_b-p_g diagram. An iso-excess demand function for good $j = g, b$ has the slope

$$\left(\frac{dp_g}{dp_b}\right)_{dZ^j=0} = -\frac{Z^j_{p_b}}{Z^j_{p_g}}$$

If ν is sufficiently small, these iso-excess demand functions are upward sloping. Now suppose that any point in this diagram, Z^b is steeper than Z^g, which holds provided that

$$Z^g_{p_g} Z^b_{p_b} > Z^g p_b Z^b_{p_g} \tag{42}$$

then a move toward higher prices for the brown good along the iso-excess demand curve for the green good, implies that the excess demand for the brown good goes down. Condition (42) can be shown to hold for ν small enough, see Lemma 7.

Observation 7. If condition (42) holds, then, for every p_c, there exist prices $p_g(p_c)$ and $p_b(p_c)$ so that

$$Z^g(p_c, p_g(p_c), p_b(p_c)) = 0 \quad \text{and} \quad Z^b(p_c, p_g(p_c), p_b(p_c)) = 0 \; .$$

To see this: By Observation 1, fix p_b at an arbitrary level and solve for the price p_g that clears the market for the green good. If at this pair of prices there is positive/ negative excess demand for the brown good, move along the iso-excess demand curve for the green good towards higher/ lower prices p_b. Eventually the excess demand for the brown good will fall/ rise to zero. This follows from the functional forms above.

Observation 8. For all p_g and p_b, the excess demand function Z^c is strictly decreasing in p_c and there is a price p_c so that $Z_c(p_c, p_g, p_b) = 0$. If the conditions detailed in Observation 7 are satisfied, one can vary p_c to clear the market for the unspecific consumption good, while keeping the markets for the green and the brown good in equilibrium. □

D.5 Proofs of Propositions 2 and 3

Recall that the excess demand function for the unspecific consumption good is

$$\mathcal{Z}^c(p_c, p_g, p_b, \cdot) \quad := \quad X_c((1+t_c)p_c, (1+t_g)p_g, (1+t_b)p_b, \cdot) - Y_c(p_c, t_{ce}) \; .$$

Also recall that the excess demand functions for the green and the brown good are

$$\mathcal{Z}^g(p_c, p_g, p_b, \cdot) \quad := \quad \frac{\nu}{(1+t_g)p_g} A_g(p_c, p_g, p_b) - Y_g(p_g, p_c, t_{ge}) \; ,$$

where

$$A_g(p_c, p_g, p_b) := \mathbf{E}\left[\alpha_g\left(h(\kappa)\tilde{\beta}\left(\frac{1+t_b}{1+t_g}\frac{p_b}{p_g}\right)\right)\ c^*(\tilde{v}(\beta, t, p), \cdot)\right] .$$

Analogously,

$$\mathcal{Z}^b(p_c, p_g, p_b, \cdot) \quad := \quad \frac{\nu}{(1+t_b)p_b} A_b(p_c, p_g, p_b) - Y_b(p_b, p_c, t_{be}) ,$$

where

$$A_b(p_c, p_g, p_b) := \mathbf{E}\left[\alpha_b\left(h(\kappa)\tilde{\beta}\left(\frac{1+t_b}{1+t_g}\frac{p_b}{p_g}\right)\right)\ c^*(\tilde{v}(\beta, t, p), \cdot)\right] .$$

and $\tilde{v}(\beta, t, p)$ is a shorthand for $\tilde{v}(\beta, (1+t_c)p_c, (1+t_g)p_g, (1+t_b)p_b)$. To allow for shifts in the preferences towards the green good, we interpret α_g and α_b henceforth as functions of $h(\kappa)\tilde{\beta}\left(\frac{1+t_b}{1+t_g}\frac{p_b}{p_g}\right)$, where $\tilde{\beta} = \beta^{\frac{1}{1-\varepsilon_X}}$ and h is an increasing function. As before, we denote by $\alpha_g'(\cdot) > 0$ the derivative of α_g with respect to this expression. We define $\alpha_b'(\cdot) < 0$ analogously.

We denote equilibrium prices for producers and consumers by p_j^* and q_j^*. We will determine how these objects change when some tax rate $\tau \in \{t_c, t_g, t_b, t_{ce}, t_{ge}, t_{be}\}$ changes. Thus we look at the system of equations

$$\mathcal{Z}^c(p_c^*(\tau), p_g^*(\tau), p_b^*(\tau), \cdot) \quad = \quad 0 ,$$

$$\mathcal{Z}^g(p_c^*(\tau), p_g^*(\tau), p_b^*(\tau), \cdot) \quad = \quad 0 ,$$

$$\mathcal{Z}^b(p_c^*(\tau), p_g^*(\tau), p_b^*(\tau), \cdot) \quad = \quad 0 .$$

and then take derivatives with respect to τ to determine how p_c^*, p_g^* and p_b^* change when τ changes.

D.5.1 Comparative statics with respect to t_c and t_{ce}.

We employ

$$\mathcal{Z}^c(p_c, p_g, p_b, \cdot) \quad := \quad X^c((1+t_c)p_c, (1+t_g)p_g, (1+t_b)p_b, \cdot) - Y^c(p_c, t_{ce}) .$$

and

$$\mathcal{Z}^c(p_c^*(\tau), p_g^*(\tau), p_b^*(\tau), \cdot) \quad = \quad 0 .$$

It follows from Lemma 6 that changes of p_g^* and p_b^* have a vanishing impact on $c^*(\cdot)$ for ν small. Thus, for ν small, we can, without loss of generality, obtain the direction in which p_c^* and q_c^* change, by treating p_b^* and p_g^* as constant. Thus, we employ

$$X_{p_c}^c(\cdot)\Big((1+t_c)\, p_{c\tau}^* + p_c^*\, \mathbf{1}(\tau = t_c)\Big) - Y_{p_c}^c(\cdot)\, p_{c\tau}^* - Y_{t_{ce}}^c(\cdot)\mathbf{1}(\tau = t_{ce}) \quad = \quad 0 . \qquad (43)$$

where $p^*_{c\tau}$ is a shorthand for the derivative of p^*_c with respect to τ, $\mathbf{1}(\tau = t_c)$ is an indicator function that takes the value 1 if $\tau = t_c$ and $\mathbf{1}(\tau = t_{ce})$ is an indicator function that takes the value 1 if $\tau = t_{ce}$.

Obviously,

$$\Big(X^c_{p_c}(\cdot)(1+t_c) - Y^c_{p_c}(\cdot)\Big)p^*_{c\tau} \quad = \quad -X^c_{p_c}(\cdot)\mathbf{1}(\tau = t_c) \ + \ Y^c_{t_{ce}}(\cdot)\mathbf{1}(\tau = t_{ce}) \ .$$

Since

$$X^c_{p_c}(\cdot)(1+t_c) - Y^c_{p_c}(\cdot) \ < \ 0 \ ,$$

$$-X^c_{p_c}(\cdot)\mathbf{1}(\tau = t_c) \ > \ 0 \, , \quad \text{and} \quad Y^c_{t_{ce}}(\cdot)\mathbf{1}(\tau = t_{ce}) \ < \ 0 \ ,$$

it follows that

$$p^*_{c\tau} \ > \ 0 \quad \text{for} \quad \tau = t_{ce} \ , \quad \text{and} \quad p^*_{c\tau} \ < \ 0 \quad \text{for} \quad \tau = t_c \ .$$

Since $q^*_c = (1+t_c)p^*_c$ it follows immediately that also

$$q^*_{c\tau} \ > \ 0 \quad \text{for} \quad \tau = t_{ce} \ .$$

To obtain also $q^*_{ct_c}$, note that for $\tau = t_c$, (43) implies that

$$X^c_{p_c}(\cdot)q^*_{ct_c} \quad = \quad Y^c_{p_c}(\cdot)\, p^*_{c\tau} \ .$$

By the previous arguments

$$X^c_{p_c}(\cdot) < 0 \quad \text{and} \quad Y^c_{p_c}(\cdot)\, p^*_{c\tau} < 0 \ .$$

Thus, we must have

$$q^*_{c\tau} \ > \ 0 \quad \text{for} \quad \tau = t_c \ .$$

D.5.2 Comparative statics of prices in the markets for the green and the brown good

It follows from Lemma 6 that changes of p^*_g and p^*_b have a vanishing impact on $c^*(\cdot)$ and hence on p^*_c for ν small. Thus, for ν small, we can, without loss of generality, obtain the direction in which p^*_g q^*_g, p^*_b and q^*_b change, by treating p^*_c and $c^*(\cdot)$ as constant. This is what we do in what follows. Hence, excess demand functions for the brown and the green good can now more simply be written as

$$\mathcal{Z}^g(p_g, p_b, t_g, t_b, t_{ge}, \kappa) \quad := \quad X^g((1+t_g)p_g, (1+t_b)p_b, \kappa) - Y^g(p_g, t_{ge}) \ ,$$

where

$$X^g((1+t_g)p_g, (1+t_b)p_b, \kappa) := \frac{\nu}{(1+t_g)p_g} A_g((1+t_g)p_g, (1+t_b)p_b, \kappa)$$

and

$$A_g((1+t_g)p_g,(1+t_b)p_b,\kappa) := \mathbf{E}\left[\alpha_g\left(h(\kappa)\,\tilde{\beta}\left(\frac{1+t_b}{1+t_g}\frac{p_b}{p_g}\right)\right)\ c^*(\cdot)\right] .$$

Analogously,

$$\mathcal{Z}^b(p_g,p_b,t_g,t_b,t_{be},\kappa) \quad := \quad X^b((1+t_g)p_g,(1+t_b)p_b,\kappa) - Y_b(p_b,t_{be}) ,$$

where

$$X^b((1+t_g)p_g,(1+t_b)p_b,\kappa) := \frac{\nu}{(1+t_b)p_b}A_b(\kappa,p_g,p_b)$$

and

$$A_b((1+t_g)p_g,(1+t_b)p_b,\kappa) := \mathbf{E}\left[\alpha_b\left(h(\kappa)\,\tilde{\beta}\left(\frac{1+t_b}{1+t_g}\frac{p_b}{p_g}\right)\right)\ c^*(\cdot)\right] .$$

To obtain comparative statics results we start with

$$\mathcal{Z}^g(p_g^*(\tau),p_b^*(\tau),t_g,t_b,t_{ge},\kappa)=0 \quad \text{and} \quad \mathcal{Z}^b(p_g^*(\tau),p_b^*(\tau),t_g,t_b,t_{be},\kappa)=0$$

and then take derivatives with respect to τ, where $\tau \in \{\kappa, t_g, t_{ge}, t_b, t_{be}\}$. Differentiating $\mathcal{Z}^g(\cdot)=0$ yields

$$\mathcal{Z}^g_{p_g}(\cdot)p^*_{g\tau} + \mathcal{Z}^g_{p_b}(\cdot)p^*_{b\tau} + \Gamma^g(\tau) = 0 \tag{44}$$

where

$$\mathcal{Z}^g_{p_g}(\cdot) := X^g_{q_g}(\cdot)(1+t_g) - Y^g_{p_g}(\cdot)$$

$$\mathcal{Z}^g_{p_b}(\cdot) := X^g_{q_b}(\cdot)(1+t_b)$$

and

$$\begin{aligned}\Gamma^g(\tau) \quad = \quad & X^g_{q_g}(\cdot)\,p_g^*\,\mathbf{1}(\tau=t_g) + X^g_{q_b}(\cdot)\,p_b^*\,\mathbf{1}(\tau=t_b)\\ & -Y^g_{t_{ge}}(\cdot)\,\mathbf{1}(\tau=t_{ge}) + X^g_\kappa(\cdot)\,\mathbf{1}(\tau=\kappa) .\end{aligned}$$

Differentiating $\mathcal{Z}^b(\cdot)=0$ yields

$$\mathcal{Z}^b_{p_g}(\cdot)p^*_{g\tau} + \mathcal{Z}^b_{p_b}(\cdot)p^*_{b\tau} + \Gamma^b(\tau) = 0 \tag{45}$$

where

$$\mathcal{Z}^b_{p_b}(\cdot) := X^b_{q_b}(\cdot)(1+t_b) - Y^b_{p_b}(\cdot)$$

$$\mathcal{Z}^b_{p_g}(\cdot) := X^b_{q_g}(\cdot)(1+t_g)$$

and

$$\begin{aligned}\Gamma^b(\tau) \quad = \quad & X^b_{q_g}(\cdot)\,p_g^*\,\mathbf{1}(\tau=t_g) + X^b_{q_b}(\cdot)\,p_b^*\,\mathbf{1}(\tau=t_b)\\ & -Y^b_{t_{be}}(\cdot)\,\mathbf{1}(\tau=t_{be}) + X^b_\kappa(\cdot)\,\mathbf{1}(\tau=\kappa) .\end{aligned}$$

From (44) and (45) we obtain:

$$p^*_{g\tau} = \frac{1}{D}\Big(\Gamma^b(\tau)\mathcal{Z}^g_{p_b}(\cdot) - \Gamma^g(\tau)\mathcal{Z}^b_{p_b}(\cdot)\Big) \tag{46}$$

and

$$p^*_{b\tau} = \frac{1}{D}\Big(\Gamma^g(\tau)\mathcal{Z}^b_{p_g}(\cdot) - \Gamma^b(\tau)\mathcal{Z}^g_{p_g}(\cdot)\Big) \tag{47}$$

where

$$D(\cdot) \quad := \quad \mathcal{Z}^g_{p_g}(\cdot)\,\mathcal{Z}^b_{p_b}(\cdot) \;-\; \mathcal{Z}^g_{p_b}(\cdot)\,\mathcal{Z}^b_{p_g}(\cdot)\,.$$

Note that $D(\cdot) > 0$ by Lemma 7.

Comparative statics with respect to t_g and t_b. Consider a change of t_g first. With

$$\Gamma^g(\tau) \quad = \quad X^g_{q_g}(\cdot)\,p^*_g$$

and

$$\Gamma^b(\tau) \quad = \quad X^b_{q_g}(\cdot)\,p^*_g$$

it follows immediately from (46) and (47) that for $\tau = t_g$,

$$p^*_{g\tau} \;<\; 0 \quad \text{and} \quad p^*_{b\tau} \;>\; 0\,.$$

Since $q^*_b = (1+t_b)p^*_b$ and since t_b is held constant, we also have

$$q^*_{b\tau} \;>\; 0\,.$$

We now show that also

$$q^*_{g\tau} \;>\; 0\,.$$

To see this, note first that

$$q^*_g = (1+t_g)p^*_g$$

implies

$$q^*_{g\tau} = \left(1 + (1+t_g)p^*_{g\tau}\frac{1}{p^*_g}\right)p^*_g \tag{48}$$

Further note that for $\tau = t_g$, (46) can be rewritten as

$$\frac{1}{p^*_g}p^*_{g\tau} \quad = \quad \frac{1}{D(\cdot)}\Big(X^b_{q_g}(\cdot)\mathcal{Z}^g_{p_b}(\cdot) - X^g_{q_g}(\cdot)\mathcal{Z}^b_{p_b}(\cdot)\Big)$$

Using

$$X^b_{q_g}(\cdot) \quad = \quad \frac{\mathcal{Z}^b_{p_g}(\cdot)}{1+t_g} \ ,$$

$$X^g_{q_g}(\cdot) \quad = \quad \frac{\mathcal{Z}^g_{p_g}(\cdot)}{1+t_g} + \frac{Y^g_{p_g}(\cdot)}{1+t_g} \ ,$$

and

$$D(\cdot) \quad := \quad \mathcal{Z}^g_{p_g}(\cdot)\,\mathcal{Z}^b_{p_b}(\cdot) \ - \ \mathcal{Z}^g_{p_b}(\cdot)\,\mathcal{Z}^b_{p_g}(\cdot) \ ,$$

this equation can be rewritten as

$$(1+t_g)\frac{1}{p^*_g}p^*_{g\tau} = -1 - \frac{\mathcal{Z}^b_{p_b}(\cdot)Y^g_{p_g}(\cdot)}{D(\cdot)} \tag{49}$$

From (48) and (49) it follows that

$$q^*_{g\tau} \quad = \quad -\frac{\mathcal{Z}^b_{p_b}(\cdot)Y^g_{p_g}(\cdot)}{D(\cdot)}p^*_g \quad > \quad 0 \ .$$

Analogous arguments can be used to show that for $\tau = t_b$,

$$p^*_{g\tau} \ > \ 0 \quad \text{and} \quad p^*_{b\tau} \ < \ 0 \ ,$$

as well as

$$q^*_{g\tau} \ > \ 0 \quad \text{and} \quad q^*_{b\tau} \ > \ 0 \ .$$

Comparative statics with respect to t_{ge} and t_{be}. Consider a change of t_{ge} first. With

$$\Gamma^g(\tau) \quad = \quad -Y^g_{t_{ge}}(\cdot)$$

and

$$\Gamma^b(\tau) \quad = \quad 0$$

it follows immediately from (46) and (47) that for $\tau = t_g$,

$$p^*_{g\tau} \ > \ 0 \quad \text{and} \quad p^*_{b\tau} \ > \ 0 \ . \tag{50}$$

Since $q^*_g = (1+t_g)p^*_g$, $q^*_b = (1+t_b)p^*_b$ and since t_g and t_b are held constant, we also have

$$q^*_{g\tau} \ > \ 0 \quad \text{and} \quad q^*_{b\tau} \ > \ 0 \ . \tag{51}$$

Analogous arguments can be used to show that (50) and (56) also hold for $\tau = t_{be}$.

Comparative statics with respect to κ. Now consider a change of κ. With

$$\Gamma^g(\tau) \;=\; X^g_\kappa(\cdot) \quad \text{and} \quad \Gamma^b(\tau) \;=\; X^b_\kappa(\cdot)$$

it follows from (46) that

$$p^*_{g\tau} = \frac{1}{D(\cdot)}\Big(X^b_\kappa(\cdot)\mathcal{Z}^g_{p_b}(\cdot) - X^g_\kappa(\cdot)\mathcal{Z}^b_{p_b}(\cdot)\Big) \tag{52}$$

With

$$X^g(\cdot) \;=\; \frac{\nu}{(1+t_g)p_g} A_g((1+t_g)p_g,(1+t_b)p_b,\kappa)$$

for

$$A_g((1+t_g)p_g,(1+t_b)p_b,\kappa) := \mathbf{E}\left[\alpha_g\left(h(\kappa)\,\tilde{\beta}\left(\frac{1+t_b}{1+t_g}\frac{p_b}{p_g}\right)\right)\; c^*(\cdot)\right] ,$$

$$X_b(\cdot) \;=\; \frac{\nu}{(1+t_b)p_b} A_b(\kappa,p_g,p_b)$$

for

$$A_b((1+t_g)p_g,(1+t_b)p_b,\kappa) := \mathbf{E}\left[\alpha_b\left(h(\kappa)\,\tilde{\beta}\left(\frac{1+t_b}{1+t_g}\frac{p_b}{p_g}\right)\right)\; c^*(\cdot)\right] .$$

and

$$\alpha_b(\cdot) \;=\; 1-\alpha_g(\cdot)$$

it follows that

$$X^b_\kappa(\cdot) \;=\; -\frac{q^*_g}{q^*_b}\, X^g_\kappa(\cdot) . \tag{53}$$

Substituting this into (52) yields

$$p^*_{g\tau} = \frac{1}{D(\cdot)}\left(-\frac{q^*_g}{q^*_b}\mathcal{Z}^g_{p_b}(\cdot) - \mathcal{Z}^b_{p_b}(\cdot)\right) X^g_\kappa(\cdot) \tag{54}$$

To prove that $p^*_{g\tau} > 0$, we now verify that

$$q^*_g\,\mathcal{Z}^g_{p_b}(\cdot) + q^*_b\,\mathcal{Z}^b_{p_b}(\cdot) \quad < \quad 0 . \tag{55}$$

Recall from the proof of Lemma 7 that

$$\mathcal{Z}^g_{p_b} \;=\; -\frac{\nu}{(1+t_g)\,p_g\,p_b}\mathbf{E}\left[\alpha_g(\cdot)\,c^*(\cdot)(\rho_{\alpha_g} - \rho^{c^*}_b(\cdot))\right]$$

and

$$\mathcal{Z}^b_{p_b} \;=\; -\frac{\nu}{(1+t_b)\,p_b^2}\mathbf{E}\left[\alpha_b(\cdot)\,c^*(\cdot)(1+\rho_{\alpha_b} - \rho^{c^*}_b(\cdot))\right] \;-\; Y^b_{p_b}(\cdot) .$$

Therefore

$$\begin{aligned}
&q_g^*\,\mathcal{Z}^g_{p_b}(\cdot) + q_b^*\,\mathcal{Z}^b_{p_b}(\cdot) \quad = \\
&-\tfrac{\nu}{p_b^*}\mathbf{E}\left[\alpha_g(\cdot)\,c^*(\cdot)(\rho_{\alpha_g} - \rho_b^{c^*}(\cdot))\right] \\
&-\tfrac{\nu}{p_b^*}\mathbf{E}\left[\alpha_b(\cdot)\,c^*(\cdot)(1+\rho_{\alpha_b} - \rho_b^{c^*}(\cdot))\right] \;-\; Y^b_{p_b}(\cdot)
\end{aligned}$$

By Lemma 6, $\rho_b^{c^*}(\cdot)$ vanishes for $\nu \to 0$. Hence, it follows that, for ν sufficiently small, we must have

$$q_g^*\,\mathcal{Z}^g_{p_b}(\cdot) + q_b^*\,\mathcal{Z}^b_{p_b}(\cdot) \quad < \quad 0\,.$$

An analogous argument can be used to show that t $p^*_{g\tau} < 0$. Since $q_g^* = (1+t_g)p_g^*$, $q_b^* = (1+t_b)p_b^*$ and since t_g and t_b are held constant, we also have

$$q^*_{g\tau} \;>\; 0 \quad \text{and} \quad q^*_{b\tau} \;<\; 0\,. \tag{56}$$

Proof of Proposition 4

Henceforth we denote the consumers' equilibrium prices by

$$q^*(\mathcal{T}) = (q_c^*(\mathcal{T}), q_g^*(\mathcal{T}), q_b^*(\mathcal{T}))$$

and the producers' equilibrium prices by

$$p^*(\mathcal{T}) = (p_c^*(\mathcal{T}), p_g^*(\mathcal{T}), p_b^*(\mathcal{T}))\,,$$

where the tax system $\mathcal{T}$ consists of the non-linear income tax schedule T_l, the emission taxes $t_e = (t_{ce}, t_{ge}, t_{be})$ and the consumption taxes $t_x = (t_c, t_g, t_b)$.

Lemma 8 *Consider a generic tax rate $\tau \in \{t_c, t_g, t_b, t_{ce}, t_{ge}, t_{be}\}$. The marginal effect of a change of τ on type θ's indirect utility is given by*

$$V_\tau(\theta,\cdot) := \tilde{v}(\beta,q)\left(s\,\Pi_\tau + \mathcal{R}_\tau - \left(\frac{dq_c^*}{d\tau}\,x_c^*(\cdot) + \frac{dq_g^*}{d\tau}\,x_g^*(\cdot) + \frac{dq_b^*}{d\tau}\,x_b^*(\cdot)\right)\right)$$

where Π_τ and $\mathcal{R}_\tau$ are, respectively, total differentials of equilibrium profits and equilibrium tax revenue.

Proof. The utility realized by a generic type θ individual in a competitive equilibrium given tax policy $\mathcal{T}$ is given by

$$V(\theta, q^*(\mathcal{T}), T_l) := c^*(\cdot)\tilde{v}(\beta,q) - k(y_l^*(\cdot),\omega) \text{ where } c^* = y_l^* - T(y_l^*) + s\Pi + R\,,$$

Note that y_l^* is a function of $\tilde{v}$ via the outer problem. The terms involving changes of labour earnings cancel, however, by the first order condition of the outer problem. By Roy's identity, the marginal effect of a change of τ on individual welfare equals

$$\tilde{v}(\beta,q)\left(s\,\Pi_\tau + \mathcal{R}_\tau - \left(\frac{dq_c^*}{d\tau}\,x_c^*(\cdot) + \frac{dq_g^*}{d\tau}\,x_g^*(\cdot) + \frac{dq_b^*}{d\tau}\,x_b^*(\cdot)\right)\right)$$

$\square$

With a generic social welfare function, the corresponding change in social welfare is given by

$$W_\tau = \mathbf{E}_\theta \left[\mathbf{g}(\tilde{v}(\beta, q), \theta) \left(s\,\Pi_\tau + \mathcal{R}_\tau - \left(\frac{dq_c^*}{d\tau}\; x_c^*(\cdot) + \frac{dq_g^*}{d\tau}\; x_g^*(\cdot) + \frac{dq_b^*}{d\tau}\; x_b^*(\cdot) \right) \right) \right]$$

Note that the revenue effect $\mathcal{R}_\tau$ is weighted by $\bar{\mathbf{g}} := \mathbf{E}[\mathbf{g}(\tilde{v}(\cdot), \beta, \omega, s)]$ in the welfare function. Analogously, the effect on equilibrium profits in sector j is weighted by

$$\bar{\mathbf{g}}_{\Pi^j} := \mathbf{E}[\mathbf{g}(\tilde{v}(\cdot), \beta, \omega, s)\; \tilde{v}(\beta, q)\; s_j]\;.$$

Below we provide a more detailed characterization of these effects. First, we note, however, that

$$-\mathbf{E}_\theta \left[\mathbf{g}(\tilde{v}(\beta, q), \theta) \left(\frac{dq_c^*}{d\tau}\; x_c^*(\cdot) + \frac{dq_g^*}{d\tau}\; x_g^*(\cdot) + \frac{dq_b^*}{d\tau}\; x_b^*(\cdot) \right) \right]$$

can be rewritten as

$$\begin{aligned}
&-\mathbf{E}\left[\mathbf{g}(\tilde{v}(\beta, q), \theta) \left(\tfrac{dq_c^*}{d\tau}\; (x_c^*(\cdot) - X_c^*(\cdot)) + \tfrac{dq_g^*}{d\tau}\; (x_g^*(\cdot) - X_g^*(\cdot)) + \tfrac{dq_b^*}{d\tau}\; (x_b^*(\cdot) - X_b^*(\cdot)) \right)\right] \\
&\quad -\bar{\mathbf{g}}\; \tfrac{dq_c^*}{d\tau}\; X_c^*(\cdot)\; - \bar{\mathbf{g}}\; \tfrac{dq_g^*}{d\tau}\; X_g^*(\cdot) - \bar{\mathbf{g}}\; \tfrac{dq_b^*}{d\tau}\; X_b^*(\cdot)\;. \\
&= -\tfrac{dq_c^*}{d\tau}\; \mathrm{Cov}(\mathbf{g}(\tilde{v}(\cdot), \theta), x_c^*(\cdot)) - \bar{\mathbf{g}}\; \tfrac{dq_c^*}{d\tau}\; X_c^*(\cdot) \qquad (57) \\
&\quad -\tfrac{dq_g^*}{d\tau}\; \mathrm{Cov}(\mathbf{g}(\tilde{v}(\cdot), \theta), x_g^*(\cdot)) - \bar{\mathbf{g}}\; \tfrac{dq_g^*}{d\tau}\; X_g^*(\cdot) \\
&\quad -\tfrac{dq_b^*}{d\tau}\; \mathrm{Cov}(\mathbf{g}(\tilde{v}(\cdot), \theta), x_b^*(\cdot)) - \bar{\mathbf{g}}\; \tfrac{dq_b^*}{d\tau}\; X_b^*(\cdot)\;.
\end{aligned}$$

Characterizing $\mathcal{R}_\tau$. Revenue can be written as

$$\begin{aligned}
\mathcal{R}(\cdot) \;=\;& (q_c^*(\mathcal{T}) - p_c^*(\mathcal{T}))X_c^*(\Pi(\cdot), \mathcal{R}(\cdot), q^*(\mathcal{T})) \\
&+(q_g^*(\mathcal{T}) - p_g^*(\mathcal{T}))X_g^*(\Pi(\cdot), \mathcal{R}(\cdot), q^*(\mathcal{T})) \\
&+(q_b^*(\mathcal{T}) - p_b^*(\mathcal{T}))X_b^*(\Pi(\cdot), \mathcal{R}(\cdot), q^*(\mathcal{T})) \\
&+\mathbf{E}[T_l(y_l^*(\cdot))] \\
&+ \sum_{j=c,b,g} t_{je}\, \mathcal{E}_j^*(\cdot)
\end{aligned}$$

For $\tau \in \{t_c, t_g, t_b\}$ we have

$$\begin{aligned}
\mathcal{R}_\tau \;=\;& \left(\tfrac{dq_c^*(\mathcal{T})}{d\tau} - \tfrac{dp_c^*(\mathcal{T})}{d\tau} \right) X_c^*(\cdot) + (q_c^*(\mathcal{T}) - p_c^*(\mathcal{T}))X_{c\tau}^* \\
&+ \left(\tfrac{dq_g^*(\mathcal{T})}{d\tau} - \tfrac{dp_g^*(\mathcal{T})}{d\tau} \right) X_g^*(\cdot) + (q_g^*(\mathcal{T}) - p_g^*(\mathcal{T}))X_{g\tau}^* \\
&\left(\tfrac{dq_b^*(\mathcal{T})}{d\tau} - \tfrac{dp_b^*(\mathcal{T})}{d\tau} \right) X_b^*(\cdot) + (q_b^*(\mathcal{T}) - p_b^*(\mathcal{T}))X_{b\tau}^* \\
&+\mathbf{E}[T_l'(y_l^*(\cdot, \theta))y_{l\tau}^*(\cdot, \theta)] \\
&+ \sum_{j=c,b,g} t_{je}\, \mathcal{E}_{j\tau}
\end{aligned}$$

where $X_{c\tau}^*$, $X_{g\tau}^*$ and $X_{b\tau}^*$ are total differentials of equilibrium quantities, $y_{l\tau}^*(\cdot, \theta)$ is the total differential of equilibrium labour supply for an individual of type θ and $\mathcal{E}_{j\tau}$ is the total differential of equilibrium emissions in sector j. For $\tau \in \{t_{ce}, t_{ge}, t_{be}\}$, there is an additional term $\mathbf{I}(\tau = \tau_{je})\mathcal{E}_j^*(\cdot)$ in this expression. Hence, we can summarize: For $\tau \in \{t_c, t_g, t_b, t_{ce}, t_{ge}, t_{be}\}$, we have

$$\begin{aligned}
\mathcal{R}_\tau \;=\; & \left(\tfrac{dq_c^*(\mathcal{T})}{d\tau} - \tfrac{dp_c^*(\mathcal{T})}{d\tau}\right) X_c^*(\cdot) + (q_c^*(\mathcal{T}) - p_c^*(\mathcal{T}))X_{c\tau}^* \\
& + \left(\tfrac{dq_g^*(\mathcal{T})}{d\tau} - \tfrac{dp_g^*(\mathcal{T})}{d\tau}\right) X_g^*(\cdot) + (q_g^*(\mathcal{T}) - p_g^*(\mathcal{T}))X_{g\tau}^* \\
& \left(\tfrac{dq_b^*(\mathcal{T})}{d\tau} - \tfrac{dp_b^*(\mathcal{T})}{d\tau}\right) X_b^*(\cdot) + (q_b^*(\mathcal{T}) - p_b^*(\mathcal{T}))X_{b\tau}^* \\
& + \mathbf{E}[T_l'(y_l^*(\cdot,\theta))y_{l\tau}^*(\cdot,\theta)] \\
& + \sum_{j=c,b,g} t_{je}\,\mathcal{E}_{j\tau} \quad + \quad \sum_{j=c,b,g} \mathbf{I}(\tau = \tau_{je})\mathcal{E}_j^*(\cdot)\;.
\end{aligned} \tag{58}$$

Characterizing $\Pi_{j\tau}$. For $\tau \in \{t_c, t_g, t_b\}$, by the envelope theorem, equilibrium profits are affected only via price changes. Thus, for $j \in \{c, g, b\}$, we have

$$\Pi_{j,\tau} \;=\; \tfrac{dp_j^*(\mathcal{T})}{d\tau} Y_j^*(\cdot) - \tfrac{dp_c^*(\mathcal{T})}{d\tau}\mathbf{E}_j[\gamma\; r^*(\cdot)]\;. \tag{59}$$

For $\tau \in \{t_{ce}, t_{ge}, t_{be}\}$, we have

$$\Pi_{j,\tau} \;=\; \tfrac{dp_j^*(\mathcal{T})}{d\tau} Y_j^*(\cdot) - \tfrac{dp_c^*(\mathcal{T})}{d\tau}\mathbf{E}_j[\gamma\; r^*(\cdot)] - \mathbf{I}(\tau = \tau_{je})\mathcal{E}_j^*(\cdot)\;. \tag{60}$$

Collecting terms. Upon collecting terms and upon making an obvious use of market-clearing conditions we find that

$$\begin{aligned}
W_\tau \;=\; & -\tfrac{dq_c^*(\mathcal{T})}{d\tau}\,\mathrm{Cov}(\mathbf{g}(\tilde{v}(\cdot),\theta), x_c^*(\cdot)) \\
& + \tfrac{dp_c^*(\mathcal{T})}{d\tau_j}\Big((\bar{\mathbf{g}}_{\Pi_c} - \bar{\mathbf{g}})(Y_c^* - \mathbf{E}[\gamma\; r_c^*]) - \;(\bar{g}_{\Pi_g} - \bar{g})\mathbf{E}[\gamma\; r_g^*]) - (\bar{g}_{\Pi_b} - \bar{g})\mathbf{E}[\gamma\; r_b^*])\Big) \\
& + \bar{\mathbf{g}}\;(q_c^*(\mathcal{T}) - p_c^*(\mathcal{T}))X_{c\tau}^*(\cdot) \\
\\
& - \tfrac{dq_g^*(\mathcal{T})}{d\tau}\,\mathrm{Cov}(\mathbf{g}(\tilde{v}(\cdot),\theta), x_g^*(\cdot)) \\
& + \tfrac{dp_g^*(\mathcal{T})}{d\tau}\;(\bar{\mathbf{g}}_{\Pi_g} - \bar{\mathbf{g}})X_g^*(\cdot) \\
& + \bar{\mathbf{g}}\;(q_g^*(\mathcal{T}) - p_g^*(\mathcal{T}))X_{g\tau}^*(\cdot) \\
\\
& - \tfrac{dq_b^*(\mathcal{T})}{d\tau}\,\mathrm{Cov}(\mathbf{g}(\tilde{v}(\cdot),\theta), x_b^*(\cdot)) \\
& + \tfrac{dp_b^*(\mathcal{T})}{d\tau}\;(\bar{\mathbf{g}}_{\Pi_b} - \bar{\mathbf{g}})X_b^*(\cdot) \\
& + \bar{\mathbf{g}}\;(q_b^*(T) - p_b^*(T))X_{b\tau}^*(\tau) \\
\\
& + \bar{\mathbf{g}}\;\mathbf{E}_\theta[T_l'(y_l^*(\cdot,\theta))y_{l\tau}^*(\cdot,\theta)] \\
& + \bar{\mathbf{g}}\sum_{j=c,b,g} t_{je}\mathcal{E}_{j\tau}^*(\cdot) \\
& + \bar{\mathbf{g}}\sum_{j=c,b,g} \mathbf{I}(\tau = \tau_{je})\mathcal{E}_j^*(\cdot)\;,
\end{aligned}$$

which proves Proposition 4.

D.6 Proof of Proposition 7

Preliminaries I. Consider an individual's outer problem under the assumption of homothetic preferences and with a wage rate that is normalized to 1: Choose c and y_l to max

$$c\,\tilde{v}(\beta,q) - k(y_l,\omega) \quad \text{s.t.} \quad c = y_l - T(y_l) + e$$

where e is a source of income that is exogenous from the individual's perspective. Thus, the optimal level of y, henceforth denoted by $y_l^*(\tilde{v}(\beta,q),\omega)$ solves

$$\max_{y_l}\;(y_l - T(y_l))\tilde{v}(\beta,q) - k(y_l,\omega)\;.$$

Note that – as an implication of Roy's identity – for any $j \in \{c,g,b\}$,

$$\frac{\partial \tilde{v}(\beta,q)}{\partial q_j} \quad = \quad -z_j^*(\beta,q)\;\tilde{v}(\beta,q)$$

where $z_j^*(\beta,q)$ is the fraction of disposable income that an individual spends on good j. Consequently, everything else equal, a price increase for a good a with a large budget share reduces the marginal utility of income more than the increase of a price with a small budget share. Hence, the former reduces labour supply more than the latter. More formally,

$$\begin{aligned} \frac{\partial y_l^*(\tilde{v}(\beta,q),\omega)}{\partial q_j} \quad &= \quad -\frac{\partial y_l^*(\cdot)}{\partial \tilde{v}(\cdot)}\,\tilde{v}(\beta,q)\;z_j^*(\beta,q) \\ &= \quad -\eta(\omega,\beta)\;y_l^*(\cdot)\;z_j^*(\beta,q)\;, \end{aligned} \tag{61}$$

is an expression that is monotonic in the budget share $z_j^*(\beta,q)$; and $\eta(\omega,\beta)$ is the elasticity of labour income y_l^* with respect to the marginal utility of disposable income $\tilde{v}(\beta,q)$ for an individual with productive ability ω and preference parameter β.

Now reconsider, the specifications

$$u(x_c,\chi(\beta x_g,x_b)) = x_c^{1-\nu}\chi(\beta x_g,x_b)^{\nu}\;.$$

and

$$\chi(\beta,x_g,x_b) = \left(\beta x_g^{1-\varepsilon_\chi} + x_b^{1-\varepsilon_\chi}\right)^{\frac{1}{1-\varepsilon_\chi}}\;.$$

that were used to proof Propositions 1 - 3, invoking the additional assumption that ν is small, so that the budget share of the unspecific consumption good is "large" relative to the budget shares of the green and the brown good. It follows from Proposition 2 that an increase of t_c and t_{ce} will push up q_c, whereas an increase of t_g, t_b, t_{ge} or t_{be} will push up q_b and q_g. By the arguments above, for ν small enough, increases of t_c and t_{ce} come with labour supply distortions that are more significant than the labour supply distortions associated with increases of t_g, t_b, t_{ge} or t_{be}.

Preliminaries II. From (16) it follows that

$$\begin{aligned} \mathcal{W}_{\tau}^{net} &= -\sum_j \frac{dq_j^*(\mathcal{T})}{d\tau} \operatorname{Cov}(\mathbf{g}(\tilde{v}(\cdot),\theta), x_j^*(\cdot)) \\ &\quad +\bar{\mathbf{g}}\ \mathbf{E}_\theta[T_l'(y_l^*(\cdot,\theta))y_{l\tau}^*(\cdot,\theta)] \end{aligned} \tag{62}$$

Using that preferences are homothetic and equation (61), we have, moreover,

$$\bar{\mathbf{g}}\ \mathbf{E}_\theta[T_l'(y_l^*(\cdot,\theta))y_{l\tau}^*(\cdot,\theta)] \ =$$

$$-\sum_j \frac{dq_j^*(\mathcal{T})}{d\tau}\ \bar{\mathbf{g}}\ \mathbf{E}_{\beta,\omega}[T_l'(y_l^*(\cdot,\theta))\ \eta(\omega,\beta)\ y_l^*(\cdot)\ z_j^*(\beta,q)]$$

Thus,

$$\mathcal{W}_{\tau}^{net} = \sum_j \frac{dq_j^*(\mathcal{T})}{d\tau}\ \bar{B}_j \tag{63}$$

where

$$\begin{aligned} \bar{B}_j &:= -\operatorname{Cov}(\mathbf{g}(\tilde{v}(\cdot),\theta), c^*(\cdot) z_j^*(\beta,q)) \\ &\quad -\bar{\mathbf{g}}\ \mathbf{E}_{\beta,\omega}[T_l'(y_l^*(\cdot,\theta))\ \eta(\omega,\beta)\ y_l^*(\cdot)\ z_j^*(\beta,q)] \end{aligned}$$

is the net benefit associated with a marginal increase of the consumer price for good j. Recall that

$$\frac{dq_j^*(\mathcal{T})}{d\tau} = \begin{cases} 1 + t_{je}\, e_{j0}, & \text{if} \quad \tau = t_j\ , \\ (1+t_j)\, e_{j0}, & \text{if} \quad \tau = t_{je}\ , \\ 0, & \text{else}\quad . \end{cases}$$

Therefore, for $\tau_c \in \{t_c, t_{ce}\}$, we have that

$$\mathcal{W}_{\tau_c}^{net} = \frac{dq_c^*}{d\tau_c}\ \bar{B}_c\ . \tag{64}$$

Likewise, for $\tau_b \in \{t_b, t_{be}\}$,

$$\mathcal{W}_{\tau_b}^{net} = \frac{dq_b^*}{d\tau_b}\ \bar{B}_b\ . \tag{65}$$

Finally, for $\tau_g \in \{t_g, t_{ge}\}$,

$$\mathcal{W}_{\tau_g}^{net} = \frac{dq_g^*}{d\tau_g}\ \bar{B}_g\ . \tag{66}$$

Proof of Statement 1. We consider the case

$$-\operatorname{Cov}(\mathbf{g}(\cdot), x_b^*(\cdot)) \quad > \quad -\operatorname{Cov}(\mathbf{g}(\cdot), x_c^*(\cdot))\ . \tag{67}$$

The reasoning for the alternative case $-\operatorname{Cov}(\mathbf{g}(\cdot), x_g^*(\cdot)) > -\operatorname{Cov}(\mathbf{g}(\cdot), x_c^*(\cdot))$ follows along the same lines. Now, the assumption that

$$\mathcal{W}_{\tau_c}^{net} = 0$$

requires that

$$- \operatorname{Cov}(\mathbf{g}(\cdot), x_c^*(\cdot)) = \bar{\mathbf{g}} \, \mathbf{E}_{\beta,\omega}[T_l'(y_l^*(\cdot,\theta)) \, \eta(\omega,\beta) \, y_l^*(\cdot)\varphi_c^*(q)] \tag{68}$$

The assumption that ν is small implies that $z_c^*(q) > z_g^*(\beta, q)$, for all β. Therefore,

$$\bar{\mathbf{g}} \, \mathbf{E}_{\beta,\omega}[T_l'(y_l^*(\cdot,\theta)) \, \eta(\cdot) \, y_l^*(\cdot) z_c^*(q)] > \bar{\mathbf{g}} \, \mathbf{E}_{\beta,\omega}[T_l'(y_l^*(\cdot,\theta)) \, \eta(\cdot) \, y_l^*(\cdot) z_b^*(\beta, q)] \tag{69}$$

Together equations (67), (68) and (69) imply that

$$-\operatorname{Cov}(\mathbf{g}(\cdot), x_b^*(\cdot)) \quad > \quad \bar{\mathbf{g}} \, \mathbf{E}_{\beta,\omega}[T_l'(y_l^*(\cdot,\theta)) \, \eta(\cdot) \, y_l^*(\cdot) z_b^*(\beta, q)] \tag{70}$$

and hence

$$\mathcal{W}_{\tau_b}^{net} > 0 \, .$$

To see that also

$$\mathcal{W}_{\tau_g}^{net} < 0$$

first define

$$B(\omega, \beta) = (\mathbf{g}(\cdot) - \bar{\mathbf{g}}) c^*(\cdot) + \bar{\mathbf{g}} \, T_l'(y_l^*(\cdot,\theta)) \, \eta(\omega,\beta) \, y_l^*(\cdot)$$

and note that

$$\mathcal{W}_{\tau_c}^{net} = 0$$

requires that

$$z_c^*(q) \mathbf{E}_{\omega,\beta}[B(\omega,\beta)] = 0 \, .$$

which can be true only if

$$\mathbf{E}_{\omega,\beta}[B(\omega,\beta)] = 0 \, .$$

Further note that

$$\begin{aligned} \bar{B}_g &= -\mathbf{E}_{\beta,\omega}[B(\omega,\beta)\varphi_b^*(\beta,q)] \\ &= -\mathbf{E}_{\beta,\omega}[B(\omega,\beta)(1 - z_g^*(\beta,q) - z_c^*(q))] \\ &= -\mathbf{E}_{\omega,\beta}[B(\omega,\beta)] - \bar{B}_b - \bar{B}_c \\ &< 0 \, , \end{aligned}$$

where the last inequality is implied by the facts that $\mathcal{W}_{\tau_c}^{net} = 0$ implies

$$\mathbf{E}_{\omega,\beta}[B(\omega,\beta)] \quad = \quad \bar{B}_c \quad = \quad 0 \, .$$

and that $\mathcal{W}_{\tau_b}^{net} > 0$ implies that

$$\bar{B}_b \quad > \quad 0 \, .$$

Proof of Statement 2. If β is a constant we can write

$$B(\beta,\omega) = B(\omega) = (\mathbf{g}(\cdot) - \bar{\mathbf{g}})c^*(\cdot) + \bar{\mathbf{g}}\; T_l'(y_l^*(\cdot,\theta))\; \eta(\omega)\; y_l^*(\cdot)$$

and, for any $j \in \{c, g, b\}$

$$z_j^*(\beta, q) = z_j^*(q)\ .$$

Consequently,

$$\bar{B}_c = -\mathbf{E}_\omega[B(\omega)]z_c^*(q)\ ,$$

$$\bar{B}_g = -\mathbf{E}_\omega[B(\omega)]z_g^*(q)\ ,$$

and

$$\bar{B}_b = -\mathbf{E}_\omega[B(\omega)]z_b^*(q)\ .$$

Thus, these expressions all have the same sign. Statement 2 follows from combining this observation with equations (64) – (66).